UNIT – Dynamics of Fluid Flow

Study of fluid flow with the Consideration of Basic Caus of the Flow i.e Force.

Taking the Fluid flow system applying NSL
↳ Momentum Eqⁿ.

Assumptions.

1) the flow is laminar
2) the flow is irrotational
3) the flow is inviscid (not having viscosity)
4) Steady flow
5) Incompressible flow

→ N-S Eqⁿ (Momentum Eqⁿ]

→ Euler's Eqⁿ of Motion (Momentum Eqⁿ)

→ Further momentum Eqⁿ is integrated
Bernoulli's Eqⁿ
[Energy Eqⁿ]
→ Conservation of Energy.

Euler's Eqⁿ of Motion along Stream lines:-

Assumptions:-
1) the flow is laminar
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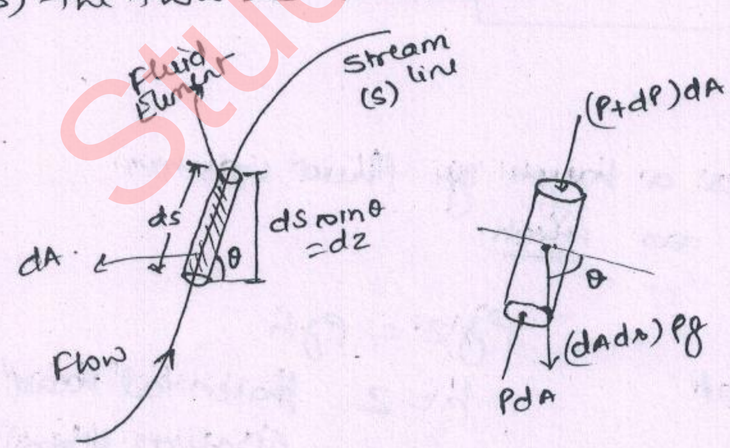


Accelaration of fluid

$$V = fn(s,t)$$

$$a = V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}$$

Applying NSL in the dir^n of flow

$$PdA - (P+dP)dA - dAds\,\rho g \sin\theta = \underbrace{dAds\,\rho}_{dm}\left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)$$

$$\boxed{\frac{dP}{\rho} + ds\left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right) + g\,dz = 0}$$ Euler's Eq^n of motion along the stream line

Steady flow $\frac{\partial V}{\partial t} \Rightarrow 0$ $V \Rightarrow f_1(x)$ only

$$\frac{\partial V}{\partial s} = \frac{dV}{ds}$$

$$\boxed{\frac{dP}{\rho} + VdV + g\,dz = 0}$$ Euler's eq^n of motion for steady flow

Integrating this eq^n

$$\int \frac{dP}{\rho} + \int VdV + \int g\,dz = \text{Const.}$$

Incompressible flow

$\therefore \rho \rightarrow$ Not changing w.r.t pressure

$$\frac{1}{\rho}\int dP + \int VdV + \int g\,dz = \text{Const.}$$

$$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{const.}$$

Energy Eq^n
Conservation of Energy
Bernoulli's Eq^n

$$\boxed{P + \underbrace{\tfrac{1}{2}\rho V^2}_{\text{K.E per Unit Volume}} + \underbrace{\rho g z}_{\text{potential Energy per Unit Volume}} = \text{Const}}$$

Pressure energy per Unit Volume (P)

$$\boxed{P + \frac{1}{2}\rho V^2 + \rho g z = \text{Const}}$$

Energy Eq^n in Head form :-

When energy is represented as a height of fluid column then that height is known as **Head.**

$P = \rho g h$

$h = \frac{P}{\rho g} \rightarrow$ Pressure head

$\rho g z = \rho g h$

$h = z$ potential head (Datum Head)

$\frac{1}{2}\rho V^2 = \rho g h$

$h = \frac{V^2}{2g}$ Velocity head
Kinetic Head
Dynamic Head

$\frac{P}{\rho g} + Z \Rightarrow$ Piezometric Head.

$\frac{P}{\rho g} + \frac{V^2}{2g} + Z \Rightarrow$ Total Head.

Energy Eq^n $P + \frac{1}{2}\rho V^2 + \rho g Z = \text{Const.}$

$$\boxed{\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Const.}}$$ Energy Eq^n in Head from

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2}$$

Energy Eq^n Between any two points (Assumption)

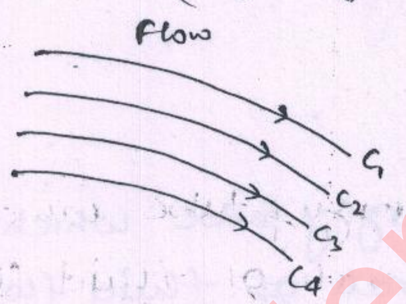
Energy Eq^n Applied in rotational fluid flow :-

total head

$$\left(\frac{P}{\rho g} + \frac{V^2}{2g} + Z\right) = \text{Const} = C$$

If flow is Irrotational

$C_1 = C_2 = C_3 = C_4 \dots = \text{Const.}$



If flow is rotational

$C_1 \neq C_2 \neq C_3 \neq C_4$ ---

Energy Eq^n can be applied in rotational fluid flows b/w the points which are on the same stream Line.

Energy Eq^n Applied in Real fluid flows :-

Real fluid flow

↳ where Viscous effects are existing

Viscous shear stress b/w the layers $\neq 0$

(Boundary layer flows)

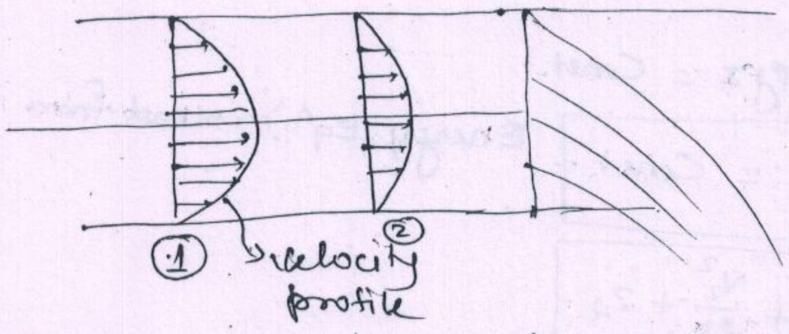
External flow → Inside as well as Outside the BL

Internal flow → Complete flow is inside the BL

(Pipe flow)

Pipe flow :-

(Boundary layer flow) (Viscous flow)



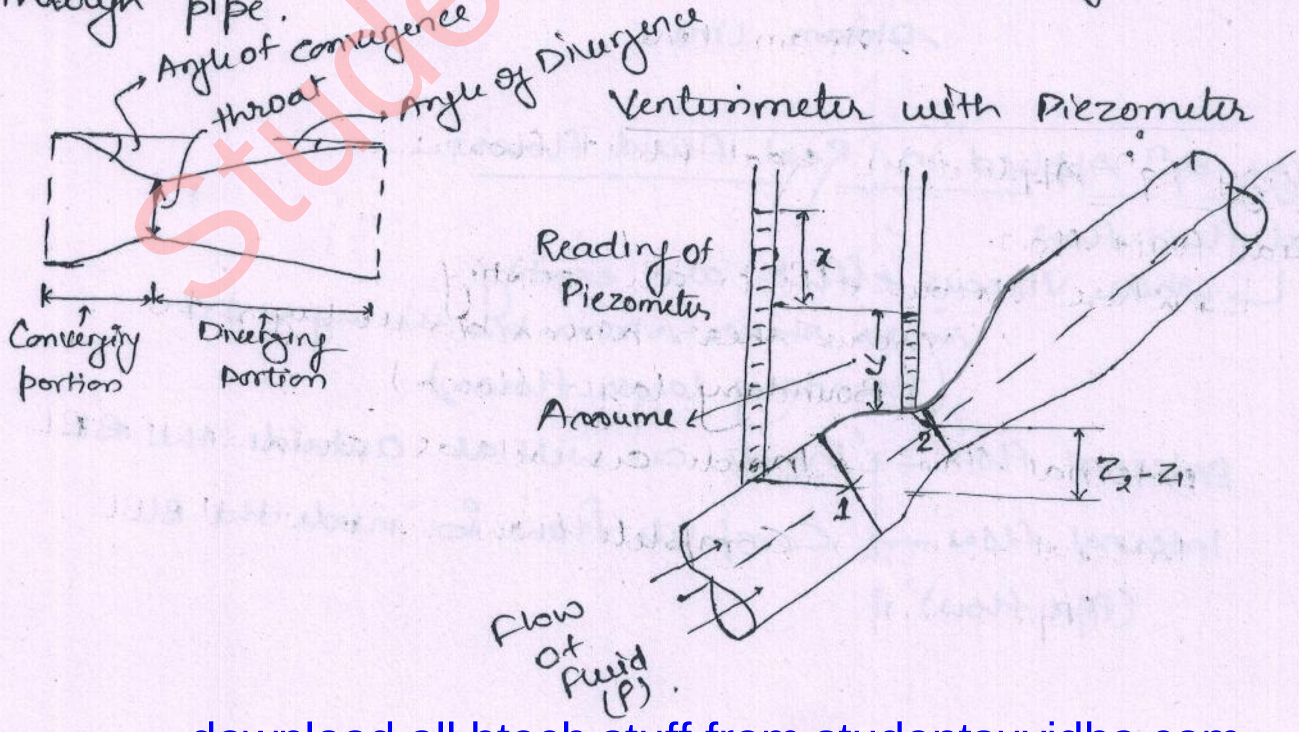
Flow in the pipes always occurs because of difference of total Head.

In the dirn of pipe flow, total Head ↓ because of head loss.

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L}$$

Flow Measurement Devices :-

1) Venturimeter :- It's a Converging Diverging tube which is basically used to measure the flow rate of fluid flowing through pipe.



Theoritical Analysis

Energy Eq^n (1, 2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{V_2^2 - V_1^2}{2g} = \underbrace{\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right)}$$

h = differential head of Venturimeter

Difference of piezometric heads b/w Normal section & throat section

$$Q_{th} = A_1V_1 = A_2V_2$$

$$\left(\frac{Q_{th}}{A_2}\right)^2 - \left(\frac{Q_{th}}{A_1}\right)^2 = 2gh.$$

$$\boxed{Q_{th} = A_1A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

$$\boxed{\dot{m}_{th} = \rho A_1A_2\sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

Now $P_1 = [x + y + (Z_2 - Z_1)]\rho g$

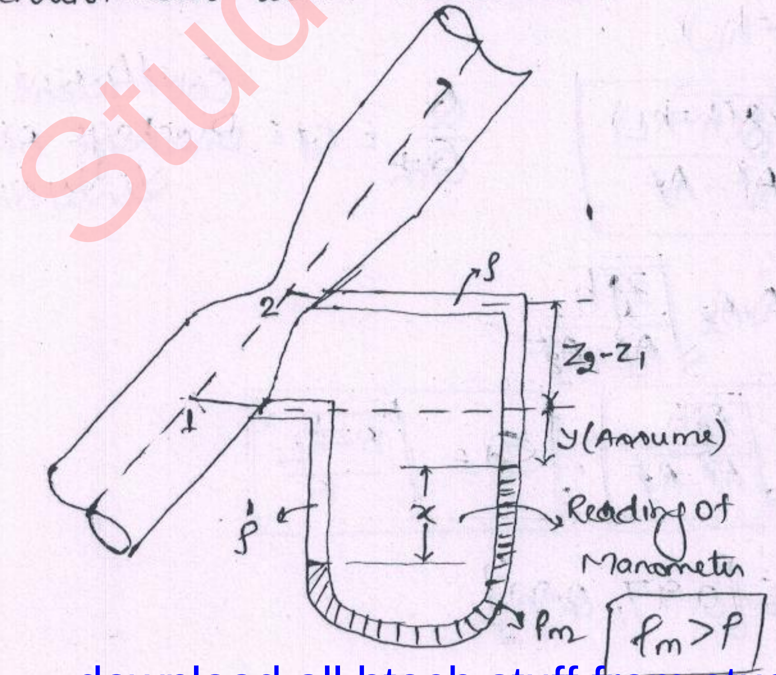
$$\frac{P_1}{\rho g} + Z_1 = x + y + Z_2.$$

and $P_2 = y\rho g \Rightarrow \frac{P_2}{\rho g} = y.$

$$\boxed{\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = x.} \quad \boxed{h = x}$$

in Venturimeter with piezometer

Venturimeter with Manometer



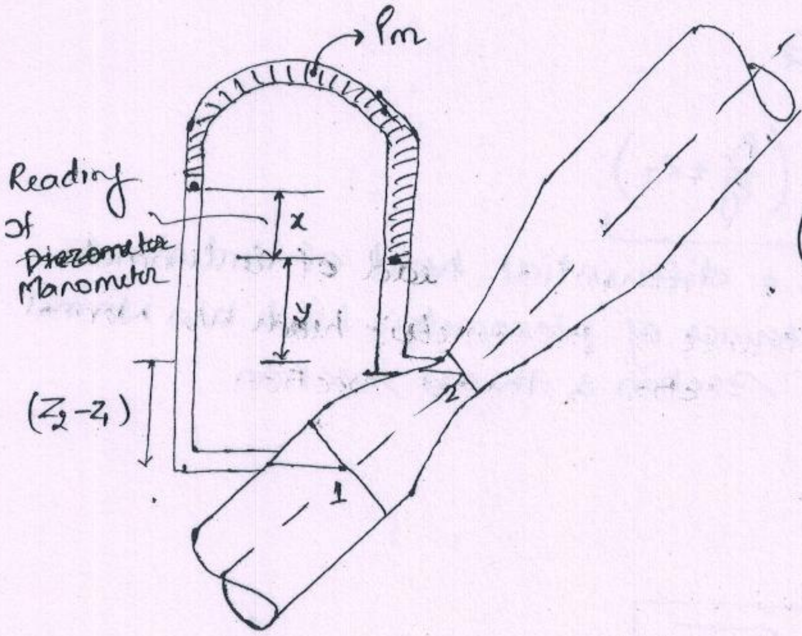
$$P_1 + (x+y)\rho g - x\rho_m g - [y + (Z_2 - Z_1)]\rho g = P_2$$

$$\frac{P_1}{\rho g} + x + y - x\frac{\rho_m}{\rho} - y - Z_2 + Z_1 = \frac{P_2}{\rho g}$$

$$\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_2}{\rho g} + Z_2\right) = h = x\left(\frac{\rho_m}{\rho} - 1\right)$$

$$\boxed{h = x\left(\frac{\rho_m}{\rho} - 1\right)}$$

Venturimeter with inverted U-tube Manometer $\boxed{\rho_m < \rho}$



$$P_1 - [(z_2 - z_1) + y + x]\rho g + x\rho_m g + y\rho g = P_2$$

$$\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = h = x\left(1 - \frac{\rho_m}{\rho}\right)$$

Here $\boxed{\rho_m < \rho_1}$

$$\boxed{h = x\left(1 - \frac{\rho_m}{\rho}\right)}$$

Actual Analysis :-

Energy Eq^n (1, 2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \underbrace{\left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right)}_{h} - h_L$$

$$\boxed{\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = h - h_L} \qquad Q = A_1 V_1 = A_2 V_2$$

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2g(h - h_L)$$

$$\boxed{Q = A_1 A_2 \sqrt{\frac{2g(h - h_L)}{A_1^2 - A_2^2}}}$$

$$\frac{Q}{Q_{th}} = C_d = \text{Coefficient of Discharge of Venturimeter}$$

$$Q = \sqrt{\frac{h - h_L}{h}}\, A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

$$\boxed{Q = C_d\, A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}} \qquad \boxed{C_d = \sqrt{\frac{h - h_L}{h}}}$$

$$C_d \in [0.97, 0.99]$$

$$\boxed{Q = C_d A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

where $C_d \rightarrow$ Coefficient of Discharge for Venturimeter

$C_d \in [0.97, 0.99]$

$h \rightarrow$ Differential head of venturimeter

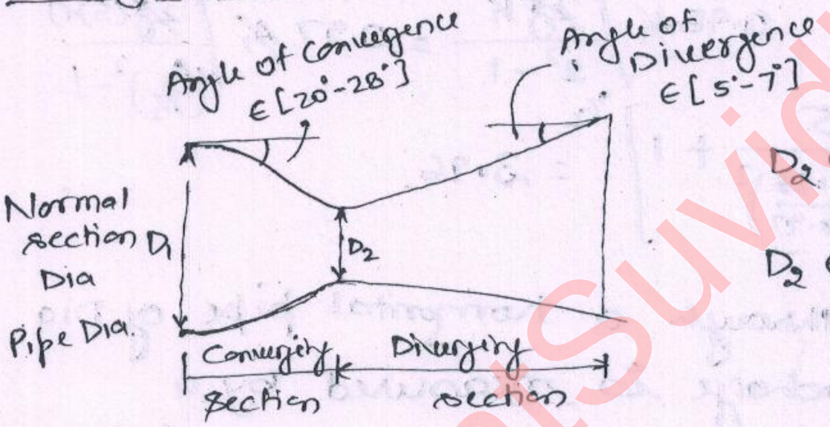
$\Rightarrow$ Difference of Piezometric heads b/w Normal section & throat section

$h = x$ (Piezometer)

$h = x\left(\frac{\rho_m}{\rho} - 1\right)$ (Manometer) $(\rho_m > \rho)$

$= x\left(1 - \frac{\rho_m}{\rho}\right)$ Manometer $(\rho_m < \rho)$

Design parameter of Venturimeter :-



$D_2 \in \left[\frac{D_1}{3}, \frac{D_1}{2}\right]$

$D_2 \in [0.33D_1, 0.5D_2]$

D_2 = throat Dia.

In Converging section

If $x \rightarrow$ dirn of flow

$\frac{\partial p}{\partial x} < 0$

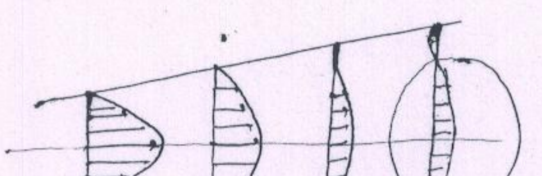
Favourable pressure Gradient

(Accer flow)

In Diverging section

$\frac{\partial p}{\partial x} > 0 \longrightarrow$ Adverse Pressure Gradient

Retarding flow



Boundary layer seperation

Note:→ In a fluid flow through pipe, if pressure is given at any section by default it is considered as gauge pressure.

Ques. One Venturimeter with the area ratio 2 and coefficient of Discharge 0.98 and the other venturimeter having $C_d = 0.97$. both are installed in the same pipe if differential head of the second venturimeter is 5 times of that of 1st Venturimeter. Find the area ratio of 2nd Venturimeter.

Ans.
$$Q = C_d A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}} = C_d A_1 \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

I V — $C_d = 0.98$, $\frac{A_1}{A_2} = 2$, h

II V — $C_d = 0.97$, $\frac{A_1}{A_2'} = ?$, $5h$

So Discharge will be same ∵ of same pipe.

$$0.98 A_1 \sqrt{\frac{2gh}{2^2-1}} = 0.97 A_1 \sqrt{\frac{2g(5h)}{\left(\frac{A_1}{A_2'}\right)^2 - 1}}$$

$$\frac{A_1}{A_2'} = \left[\frac{5}{\left(\frac{0.98}{0.97}\right)^2} + 1\right]^{1/2} = 3.96.$$

Ques. Water is flowing through a horizontal pipe of Dia 30 cm and its discharge is measured by a Venturimeter installed in the pipe which is having 15cm throat Dia the pressure of the water at the inlet of the venturimeter is 100KPa and 40cm of Mercury Vaccume If the 4% of the differential head is lost in the converging section of the venturimeter find the discharge of the water through the pipe.

Ans
$D_1 = 30\text{cm}$
$D_2 = 15\text{cm}$
$P_1 = 100\text{KPa} = 10^5\text{ Pa}$
$P_2 = 40\text{ cm (Hg) (Vacuum)}$
$= -0.4 \times 13600 \times 9.81 \text{ Pa}$

$$h = \left(\frac{P_1}{\rho g} + z_1\right) - \left(\frac{P_2}{\rho g} + z_2\right) = \frac{10^5 - (-0.4 \times 13600 \times 9.81)}{1000 \times 9.81} \text{ meter}$$

$h_L = 0.04h$

$$C_d = \sqrt{\frac{h-h_L}{h}} = \sqrt{\frac{h-0.04h}{h}} = \sqrt{0.96} = 0.979.$$

$$Q = C_d A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

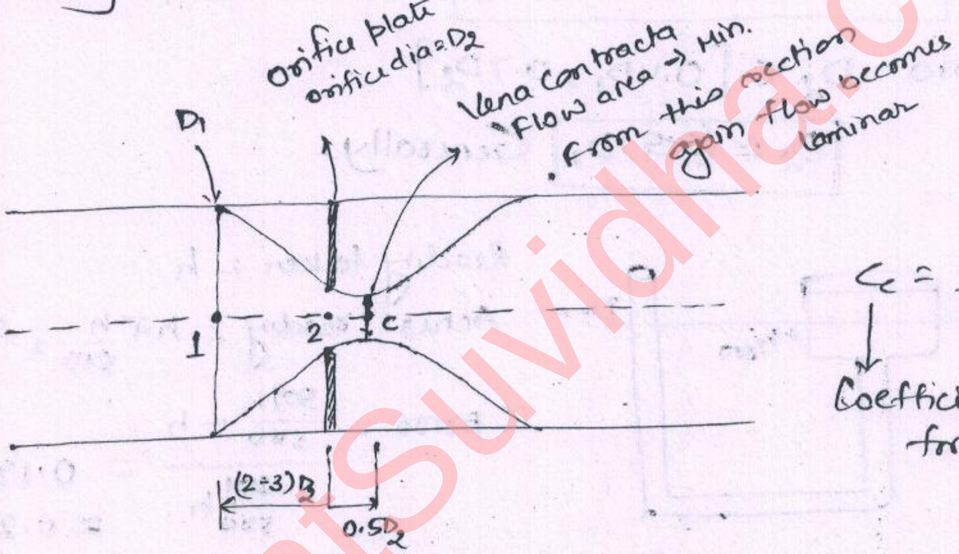
$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.3)^2$

$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.15)^2$

$= \cancel{0.259 \, m^3/s}.$

$= 0.31 \, m^3/s.$

2) Orifice meter:- It is very cheap device which is basically used for the flow measurements



$C_c = \frac{A_c}{A_2}$

Coefficient of Contraction for orifice.

Energy Eq^n (1, c)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_c}{\rho g} + \frac{V_c^2}{2g} + Z_c$$

$$\frac{V_c^2 - V_1^2}{2g} = \underbrace{\left(\frac{P_1}{\rho g} + Z_1\right) - \left(\frac{P_c}{\rho g} + Z_c\right)}_{h}$$

$h \Rightarrow$ Differential head of orifice meter

Continuity Eq^n 2, c

$A_2 V_2 = A_c V_c$

$$V_c = \frac{A_2 V_2}{A_c} = \frac{V_2}{(A_c/A_2)} = \frac{V_2}{C_c}$$

$$V_c^2 - V_1^2 = 2gh$$

$$\left(\frac{V_2}{C_c}\right)^2 - V_1^2 = 2gh$$

$$\frac{Q^2}{A_2^2 C_c^2} - \frac{Q^2}{A_1^2} = 2gh$$

$$Q^2(A_1^2 - A_2^2 C_c^2) = A_1^2 A_2^2 C_c^2 \, 2gh$$

$$Q = C_c A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2 C_c^2}}$$

$$Q = \underbrace{\boxed{C_c \sqrt{\frac{A_1^2 - A_2^2}{A_1^2 - A_2^2 C_c^2}}}}_{C_d \text{ of orific meter}} A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

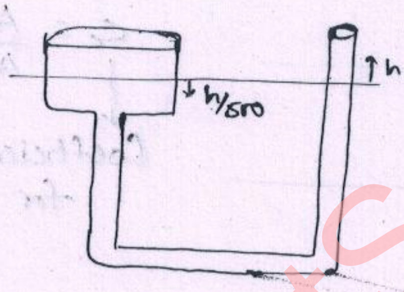
$C_d \in [0.62, 0.67]$

$$\boxed{Q = C_d A_1 A_2 \sqrt{\frac{2gh}{A_1^2 - A_2^2}}}$$

Orifice Dia $D_2 \in [0.4D_1, 0.7D_2]$

$\boxed{D_2 \simeq (0.5)D_1}$ Generally.

Ques 14
WB 12.

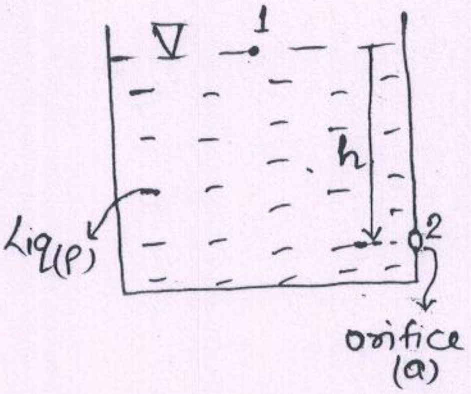


Reading taken = h

Actual reading = $h + \frac{h}{500} = \frac{501h}{500}$

$$\% \text{ Error} = \frac{\frac{501h}{500} - h}{\frac{501}{500}h} = 0.199\% \simeq 0.2\%$$

Flow through orifices :-



(point)

Point 2 [Highly turbulent Zone]
just outside the orifice

Energy Eq^n b/w 1 & 2.

$$\frac{P_{atm}}{\rho g} + \frac{V_1^2}{2g} + h = \frac{P_{atm}}{\rho g} + \frac{V_2^2}{2g} + 0$$

$$A_1 V_1 = a_1 V_2 \qquad \frac{a_1}{A_1} \simeq 0$$

$$V_1 = \frac{a_1 V_2}{A_1} \qquad V_1 \to 0$$

Actual velocity of Efflux   so $V_2^2 = 2gh$.

$$\boxed{V_{2th} = \sqrt{2gh}}$$

$V_{act} < V_{th}$.

$\frac{V}{V_{th}} < C_V \rightarrow$ Coefficient of velocity

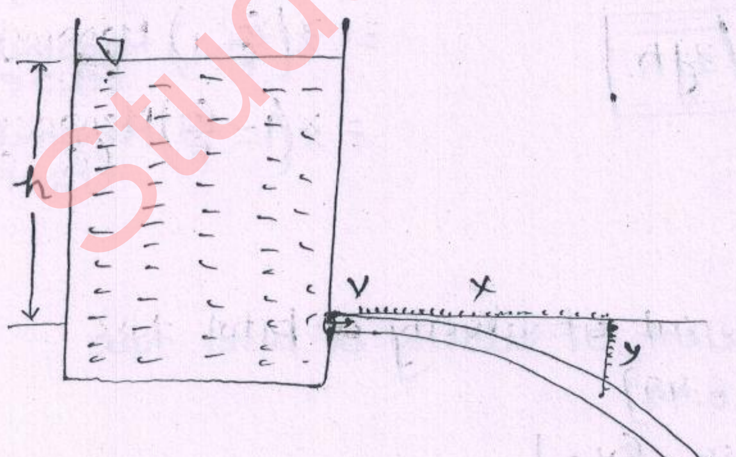
$$\boxed{V = C_V\sqrt{2gh}} \quad \text{where } 0 < C_V < 1$$

Discharge :- $Q_{th} = a_2\sqrt{2gh}$.

$\frac{Q}{Q_{th}} = C_d \rightarrow$ Coefficient of Discharge

$$Q = C_d Q_{th} = C_d a_2 \sqrt{2gh}$$

where $\boxed{C_d = C_c C_V}$



$$x = Vt$$

$$y = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{x}{V}\right)^2$$

$$V^2 = g\frac{x^2}{2y}$$

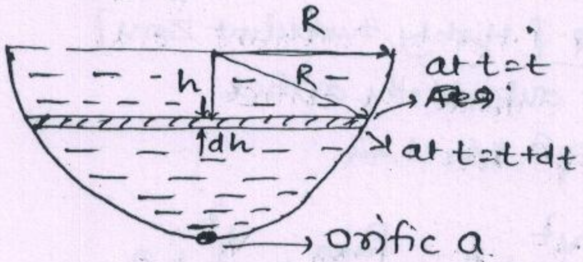
$$\boxed{V = x\sqrt{\frac{g}{2y}}}$$

$V_{th} = \sqrt{2gh}$

$$C_V = \frac{V}{V_{th}} = \frac{x\sqrt{\frac{g}{2y}}}{\sqrt{2gh}}$$

$$\boxed{C_V = \frac{x}{2\sqrt{yh}}}$$

Ques find the time taken for empty the vessel



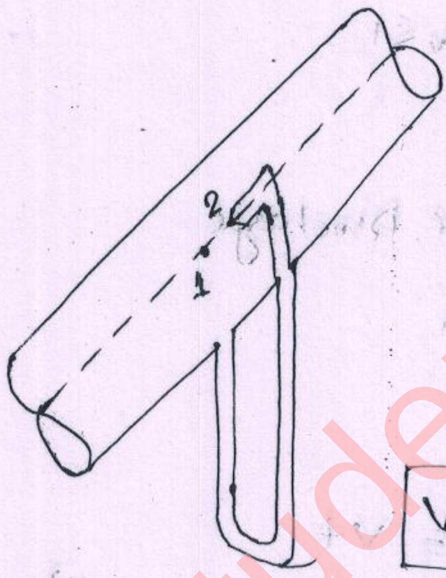
Ans

$$C_d a \sqrt{2g(R-h)}\, dt = \pi(R^2-h^2)dh.$$

$$\int_0^R \frac{(R^2-h^2)}{\sqrt{R-h}} dh = \frac{C_d a \sqrt{2g}}{\pi} \int_0^T dt$$

$$T = ?$$

Pitot tube :→ It is a device which is basically used to measure local velocities in the pipe flow.



Energy Eq^n (1,2)

$$\frac{P_1}{\rho g} + \frac{V_{1th}^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{V_{1th}^2}{2g} = \underbrace{\left(\frac{P_2}{\rho g} + Z_2\right) - \left(\frac{P_1}{\rho g} + Z_1\right)}_{h}$$

$h = x$ (Piezometer)

$= x\left(\frac{\rho_m}{\rho} - 1\right)$ Manometer ($\rho_m > \rho$)

$= x\left(1 - \frac{\rho_m}{\rho}\right)$ Manometer ($\rho_m < \rho$)

$$\boxed{V_{1th} = \sqrt{2gh}}$$

It has been seen

$V_1 = C_v \sqrt{2gh}$

↳ Coefficient of velocity of pitot tube

$C_v \in [0.98, 0.99]$

If it is not given $C_v = 1$

Note!- In fluid flow system the points where the velocity of flow is zero and known as stagnation points.

$$(\text{Kinetic Head})_{th} = \text{Stagnation piezometer head} - \text{Static piezometer Head}$$

If pipe is Horizontal.

$$(\text{kinetic Head})_{th} = \text{Stagnation pressure Head} - \text{Static pressure Head}$$

Ques:- Water is flowing through the horizontal pipe of dia 20 cm. the stagnation pressure head and the static pressure head at the Centre line of the pipe are 150 kPa and 50 cm of Hg column. If the mean velocity is half of the maximum velocity find the flow rate of water through the pipe. (take C_v for pitot tube = 0.98)

Ans $P_2 = 150\,\text{kPa} \Rightarrow \frac{150 \times 1000}{1000 \times 9.81} = \frac{P_2}{\rho g} = 15.29\,m$

$P_1 = 50\,cm\,(Hg)$

$= (0.5 \times 13600 \times 9.81)\,Pa$

$$\frac{P_1}{\rho g} = \frac{0.5 \times 13600 \times 9.81}{1000 \times 9.81} = 6.80\,\text{meter}$$

$$\left(\text{Stagnation pressure Head}\right) - \left(\text{Static pressure Head}\right) = (\text{Kinetic Head})_{th}$$

$$15.29 - 6.80 = \frac{V_{1th}^2}{2g}$$

$$\boxed{(V_{1th}) = 12.9\,m/sec}$$ at Centerline

$$(V_1)_{\text{centre line}} = 0.98 \times 12.9 = 12.65\,m/s.$$

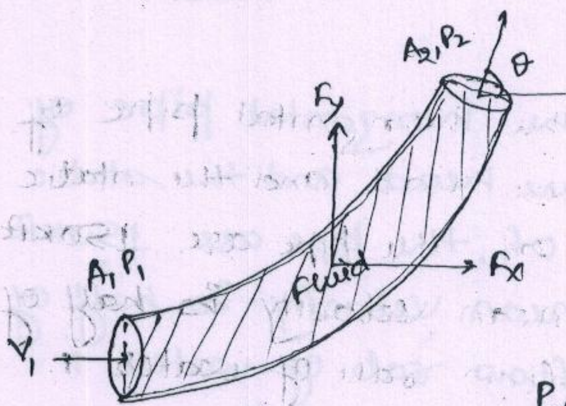
$$(V_1)_{mean} = \frac{12.65}{2} = 6.325\,m/s$$

$$Q = \frac{\pi}{4}(0.2)^2 \times 6.325 = 0.198\,m^3/s.$$

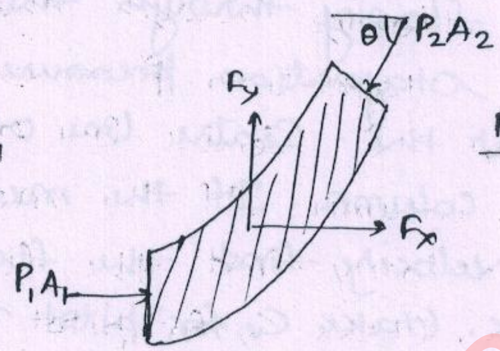
Forces on pipe Bends :-

Horizontal Bend.

Aim :- force exerted by the fluid on the pipe Bend



F_x, F_y force exerted by Bend on fluid.



Fluid system

x dim.

$$P_1 A_1 + F_x - P_2 A_2 \cos\theta = (\dot{m} V_2 \cos\theta - \dot{m} V_1)$$

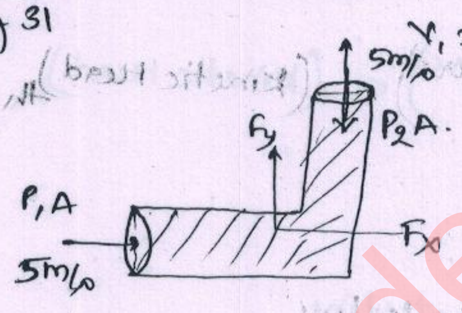
Y dim

$$F_y - P_2 A_2 \sin\theta = \dot{m} V_2 \sin\theta - 0$$

Ques. 3
Pg 31

$D = 0.3\,m \quad \theta = 90°$ Bend

$V_1 = V_2 = 5\,m/s \quad P_1 = P_2 = 4000$ Pa.



Y dim

$$F_y - P_2 A = \dot{m}\, 5$$

$$F_y = P_2 A + \dot{m} 5$$

$$= 4000 \times \frac{\pi}{4}(0.3)^2 + 1000 \times \frac{\pi}{4}(0.3)^2 \times 5 \times 5$$

$$\boxed{F_y = 2.049\ KN}$$

VORTEX FLOW :- When Certain mass of fluid is rotating w.r. to Certain axis then such a flow of fluid mass is known as Vortex flow.

there are two different types of Vortex flows.

1) Free Vortex :- When in the Vortex flow, there is no external torque requirement such a flow is known as free Vortex flow.

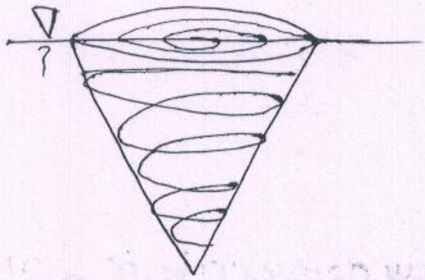
$\vec{\tau}_{ext} = 0$ → Angular momentum

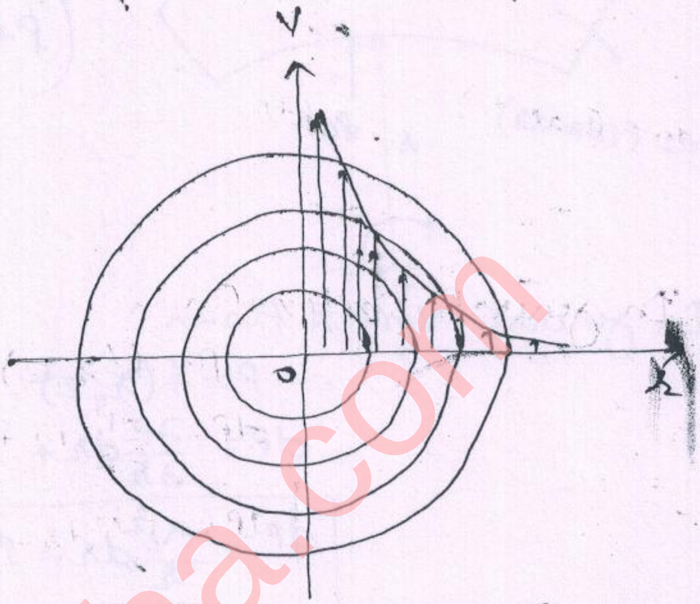
$\frac{d\vec{J}}{dt} = 0 \Rightarrow \vec{J} = \text{Const}.$

$mvr = \text{Const}.$

$\boxed{Vr = \text{Const}}$

$\boxed{V \propto \frac{1}{r}}$

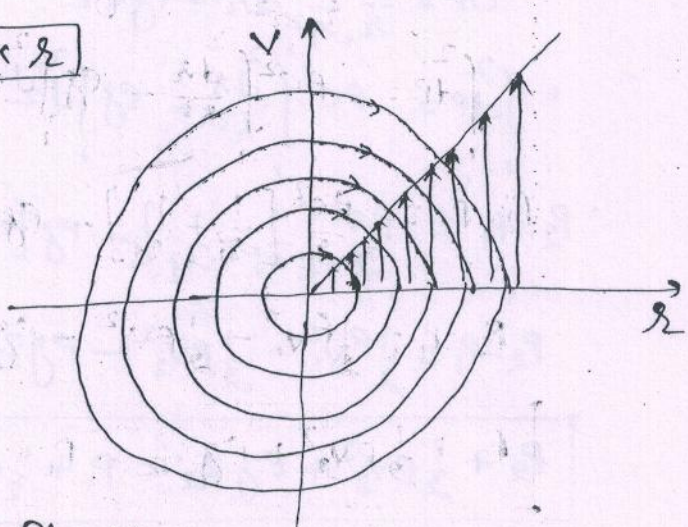




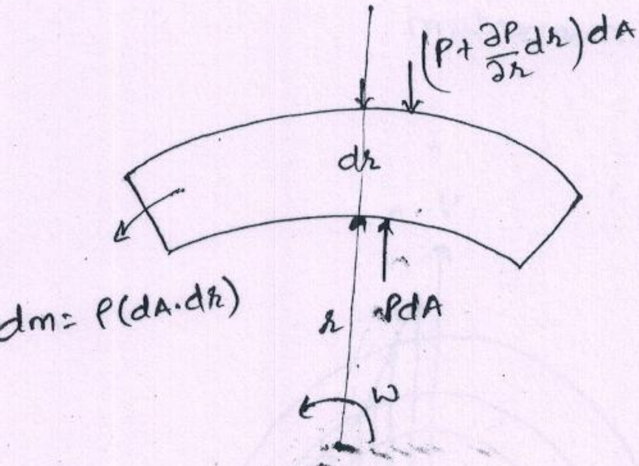
2) Forced Vortex Flow :- When external torque is Continuously applied to rotate the fluid mass with the Constant angular velocity then such a flow is known as forced Vortex flow.

$\omega = \text{Const}.$

$\frac{V}{r} = \text{Const}$ $\boxed{V \propto r}$



Fundamental Eq^n of Vortex Flow :-



$\cancel{\left(PA\frac{\partial P}{\partial r}A\right)}$

$$\left(P+\frac{\partial P}{\partial r}dr\right)dA - PdA = \rho\, dA\, dr\,\frac{V^2}{r}$$

$$\boxed{\frac{\partial P}{\partial r} = \frac{\rho V^2}{r}} > 0 \quad (P\uparrow \text{ in radial outward dirn})$$

In general Vortex flow

$$P = f(r, z)$$

$$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial z}dz.$$

$$\boxed{dP = \frac{\rho V^2}{r}dr - \rho g\, dz}$$ Fundamental eqn of Vortex flow.

In free Vortex flow

$$dP = \frac{\rho V^2}{r}dr - \rho g\, dz.$$

In free Vortex $V \propto \frac{1}{r} \Rightarrow V = \frac{C}{r}$

$$dP = \frac{\rho}{r}\frac{C^2}{r^2}dr - \rho g\, dz.$$

$$\int_1^2 dP = \rho C^2\int_1^2 \frac{dr}{r^3} - \rho g\int_1^2 dz$$

$$P_2 - P_1 = \frac{\rho C^2}{2}\left[\frac{1}{r_1^2} - \frac{1}{r_2^2}\right] - \rho g\left[z_2 - z_1\right].$$

$$P_2 - P_1 = \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2 - \rho g z_2 + \rho g z_1.$$

$$\boxed{P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 = P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1}$$

Bernoulli's eqn is valid in free Vortex flow b/w any two points

Free Vortex flow $\Rightarrow$ Irrotational Vortex.

In forced Vortex flow

$$dP = \frac{\rho V^2}{r}dr - \rho g\, dz.$$

In forced Vortex

$$V \propto r \Rightarrow V = \omega r$$

$$\int_1^2 dP = \int_1^2 \rho r w^2 dr - \int_1^2 \rho g dz$$

$$P_2 - P_1 = \frac{\rho w^2}{2}(r_2^2 - r_1^2) - \rho g z_2 + \rho g z_1$$

$$\boxed{P_2 - \frac{1}{2}\rho V_2^2 + \rho g z_2 = P_1 - \frac{1}{2}\rho V_1^2 + \rho g z_1}$$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 \neq P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2.$$

Bernoulli's theorem in general in forced vortex flow is not valid.

Forced Vortex flow ⇒ Rotational flow/Vortex.

Eqⁿ of free surface in forced vortex flow:-

In Vortex flow

$$dP = \frac{\rho V^2}{r} dr - \rho g dz$$

In forced vortex. $V = r w$

$$dP = \rho r w^2 dr - \rho g dz$$

for free surface $P = \text{Const}$.

$dP = 0$

$$\rho r w^2 dr - \rho g dz = 0$$

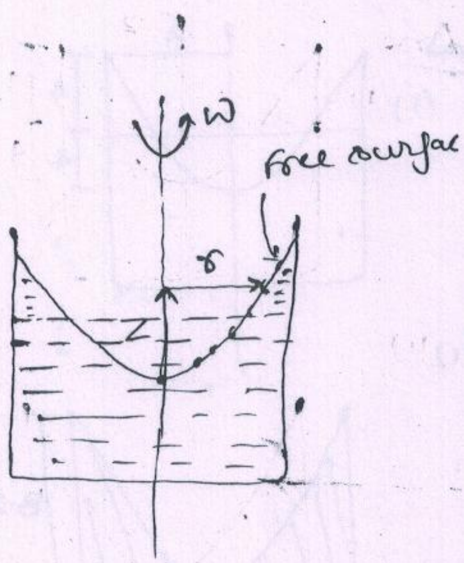
$$\int \rho g dz = \int \rho r w^2 dr$$

$$Z = \frac{r^2 w^2}{2g} + C$$

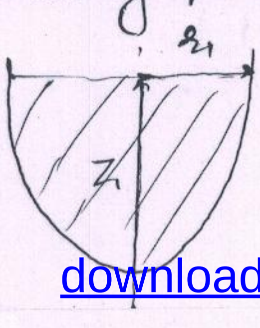
Free surface is parabola

At $r = 0,\ Z = 0,\ C = 0$

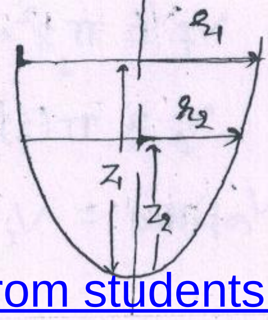
$$\boxed{Z = \frac{r^2 w^2}{2g}}$$



Volume of paraboloid :-

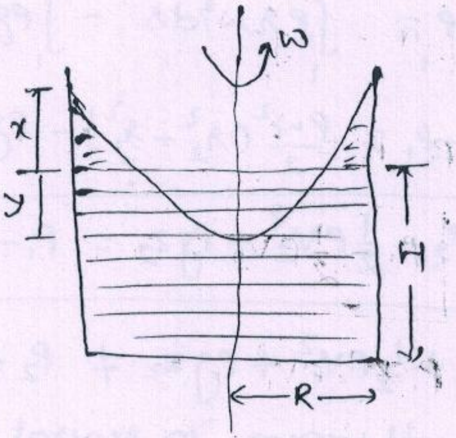


$$V_1 = \frac{1}{2}\pi r_1^2 Z_1$$



$$V_1 - V_2 = \frac{1}{2}\pi r_1^2 Z_1 - \frac{1}{2}\pi r_2^2 Z_2$$

Conservation of Volume:-



$V_i = \pi R^2 H$

$V_f = \pi R^2 (H+x) - \frac{1}{2}\pi R^2 (x+y)$

$= \pi R^2 \left(H + x - \frac{x}{2} - \frac{y}{2}\right)$

$V_f = \pi R^2 \left(H + \frac{x}{2} - \frac{y}{2}\right)$

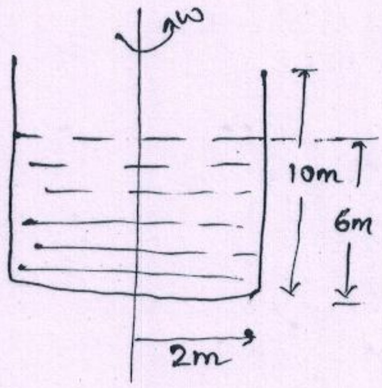
Conservation of ~~mass~~ volume.

$$V_i = V_f$$

$$\pi R^2 H = \pi R^2 \left(H + \frac{x}{2} - \frac{y}{2}\right)$$

$$\boxed{x = y}$$

Ques:-



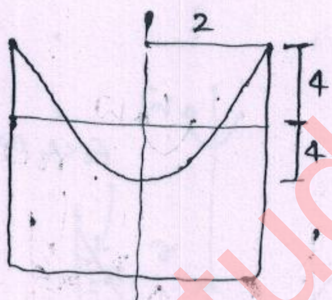
1) Find the w_{max} such that water does not spill out from the vessal.

2) If $w = 6.35$ rad/s
$V_{spilled} = ?$

3) If $w = 8$ rad/s
$V_{spilled} = ?$

Ans

(1)

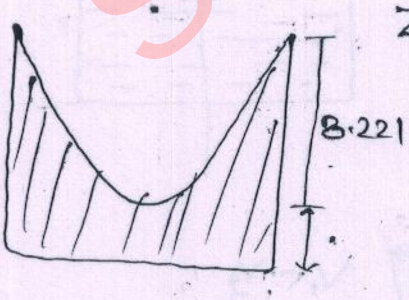


$$Z = \frac{r^2 w^2}{2g}$$

$$8 = \frac{2^2 w_{max}^2}{2 \times 9.81}$$

$$w_{max} = 6.264 \text{ rad/sec}$$

(ii)



$$Z = \frac{r^2 w^2}{2g} = \frac{2^2 (6.35)^2}{2 \times 9.81}$$

$$Z = 8.221 \text{ m}$$

$$V_f = \pi 2^2 \times 10 - \frac{1}{2}\pi (2)^2 \times 8.221$$

$$V_i = \pi (2)^2 \times 6$$

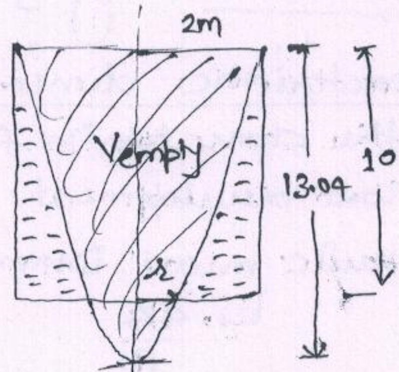
$$V_{spilled} = V_i - V_f$$

Ans 3) $w = 8$ rad/s

$V_{spilled} = ?$

$$Z = \frac{r^2 w^2}{2g} = \frac{2^2 \times 8^2}{2 \times 9.81}$$

$$Z = 13.04\,m$$



$$(3.04) = \frac{r^2 \times 8^2}{2 \times 9.81}$$

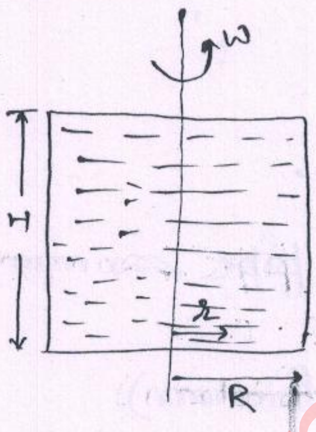
$$r = 0.965\,m$$

$$V_{empty} = \left[\frac{1}{2}\pi(2)^2 \times 13.04 - \frac{1}{2}\pi \times (0.965)^2 \times 3.04\right]$$

$$V_f = \pi(2)^2 \times 10 - V_{empty}$$

$$V_{spilled} = V_i - V_f$$

Ques

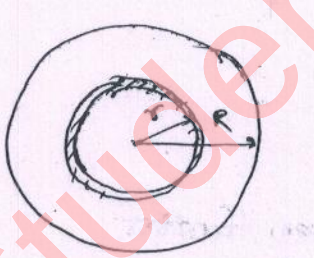


$$F_1 = (\pi R^2)(\rho g H)$$

$$\frac{\partial P}{\partial r} = \frac{\rho V^2}{r} = \rho r w^2$$

$$\int dP = \int \rho r w^2 dr$$

$$P = \frac{\rho r^2 w^2}{2}$$



$$F_2 = \int_0^R P(2\pi r)dr$$

$$= \int_0^R \frac{\rho r^2 w^2}{2}(2\pi r)dr$$

$$F_2 = \frac{\pi \rho w^2 R^4}{4}$$

so $F_{Bottom} = \dfrac{\pi \rho w^2 R^4}{4} + \rho g H(\pi R^2)$

$$\boxed{F_{Bottom} = \pi R^2\left(\rho g H + \frac{\rho w^2 r^2}{4}\right)}$$

PIPE FLOWS:- Highly viscous flows. (Internal flows)

Characteristic dimension,
↳ the dimension in flow which plays most important role in flow development

Hydraulic mean Diameter
↳ $\frac{4A}{P}$

for circular pipe $A = \frac{\pi}{4} D^2$, $P = \pi D$

so $\frac{4A}{P} = \frac{4 \times \frac{\pi}{4} D^2}{\pi D} = D$

Reynold's No.

$$Re_D = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

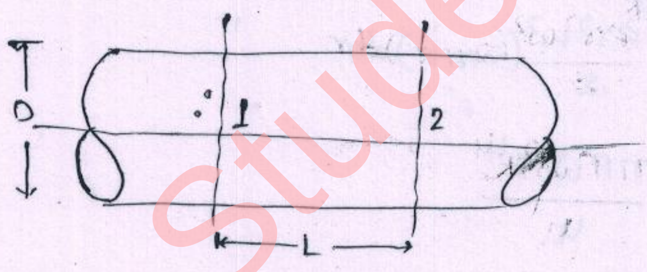
If $Re_D < 2000$ (laminar)
$Re_D > 4000$ (Turbulent)

Energy losses in the pipe flows:-

when the fluid is flowing through the pipe so many Energy losses are there in a flow

I) Major Energy Loss:- (Head loss due to friction)

1) Darcy-Weisback formula



$$h_f = \frac{fLV^2}{2gD}$$

f = Friction factor
$f = 4f'$
f' = Darcy's coefficient of friction

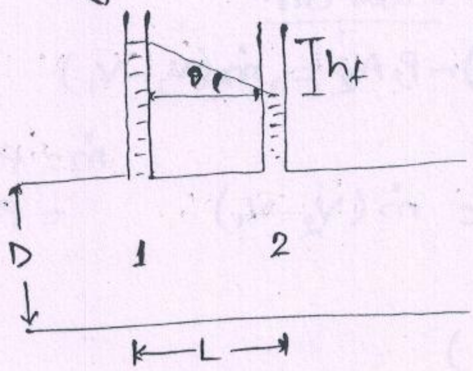
If $Re < 2000$ (Laminar)

$$f' = \frac{16}{Re_D}$$

If $Re > 4000$ (Turbulent)

$$f' = \frac{0.079}{(Re_D)^{1/4}}$$

2) Chezy's formula:-



$i = \tan\theta = \frac{h_f}{L}$

Chezy's formula

$V = C\sqrt{mi}$

m = Hydraulic mean depth.

$m = \frac{A}{P} = \frac{D}{4}$

$V = C\sqrt{\frac{D}{4} \cdot \frac{h_f}{L}}$

$V^2 = \frac{C^2 D}{4} \frac{h_f}{L}$

$$\boxed{h_f = \frac{4V^2 L}{C^2 D}}$$

C = Chezy's Constant.

Que. what should be the value of chezy's constant so that the head loss due to friction obtained from both formula is exactly same.

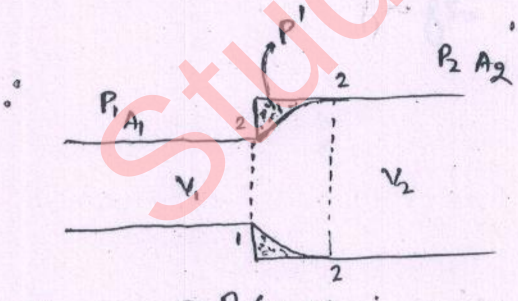
Ans

$$(h_f)_{D-W} = \frac{fLV^2}{2Dg} \qquad (h_f)_C = \frac{4V^2 L}{C^2 D}$$

$$\frac{fV^2\cancel{L}}{2g\cancel{D}} = \frac{4V^2\cancel{L}}{C^2\cancel{D}} \Rightarrow \boxed{C = \sqrt{\frac{8g}{f}}}$$

b) Minor Energy losses:-

(i) Head loss due to sudden Enlargement in pipes.



Experimently it has been seen that

$P' \simeq P_1$

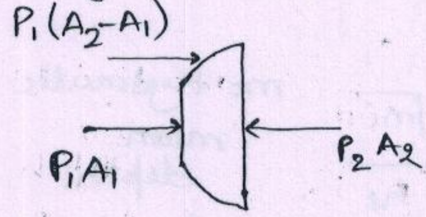
Energy Eq^n (1,2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{Le}$$

$$h_{Le} = \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} \quad \text{——— (1)}$$

taking the fluid as a system

NSL Flow dirn



$$P_1A_1 + P'(A_2-A_1) - P_2A_2 = \dot{m}(V_2-V_1)$$

$$P' \simeq P_1$$

$$\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$$

$$\cancel{P_1(A_2-A_1)} = \dot{m}(V_2-V_1)$$

$$A_2(P_1-P_2)$$

$$A_2(P_1-P_2) = \rho A_2 V_2 (V_2 - V_1)$$

$$\frac{P_1-P_2}{\rho g} = \frac{V_2^2 - V_1V_2}{g} \quad -(2)$$

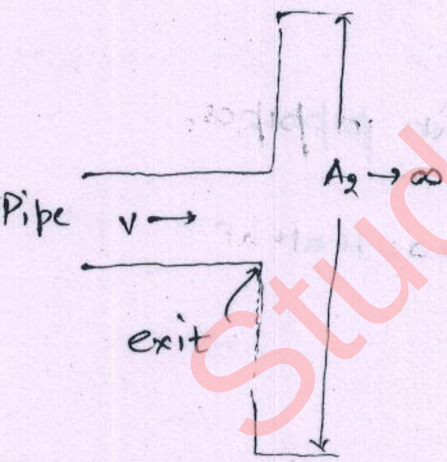
By (1) & (2)

$$h_{L_e} = \frac{V_2^2 - V_1V_2}{g} + \frac{V_1^2 - V_2^2}{2g}$$

$$h_{L_e} = \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g}$$

$$\boxed{h_{Le} = \frac{(V_1-V_2)^2}{2g}}$$

(2) Head loss at the exit of pipe :-



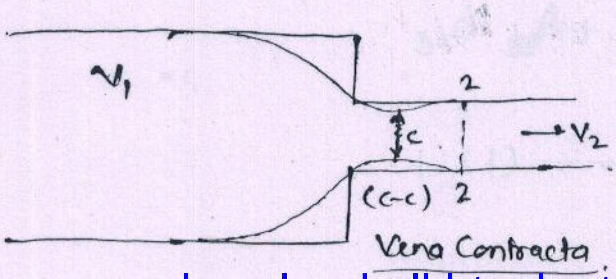
$$A_1V_1 = A_2V_2$$

$$V_2 \rightarrow 0$$

$$h_{L_{exit}} = \frac{(V_1-V_2)^2}{2g} = \frac{V^2}{2g}$$

$$\boxed{h_{L_{exit}} = \frac{V^2}{2g}}$$

Head loss due to sudden Contraction :-



$$h_{L_c} = \frac{(V_c - V_2)^2}{2g}$$

$$h_{L_c} = \frac{\left(\frac{V_2}{C_c} - V_2\right)^2}{2g}$$

$$h_{Lc} = \left(\frac{1}{C_c} - 1\right)^2 \frac{V_2^2}{2g}$$

$$A_c V_c = A_2 V_2$$

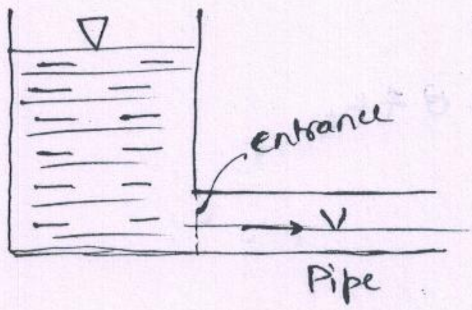
$$V_c = \frac{V_2}{(A_c/A_2)} = \frac{V_2}{C_c}$$

If C_c is not given

then $\left(\frac{1}{C_c} - 1\right)^2 \approx 0.5$

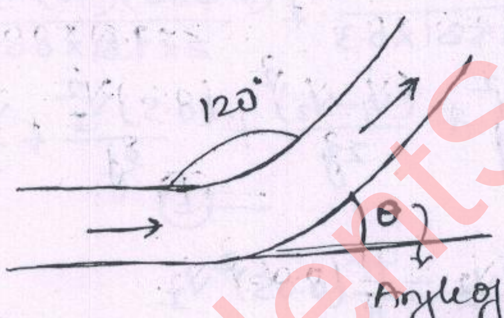
$$\boxed{h_{Lc} = (0.5)\frac{V_2^2}{2g}}$$

4. head loss at the entry of pipe:-



$$\boxed{h_{L_{entry}} = 0.5\frac{V^2}{2g}}$$

5. head ~~of~~ loss for the pipe Bends:-



$$\boxed{h_{L_{Bend}} = K\frac{V^2}{2g}}$$

K → loss coefficient for pipe Bend

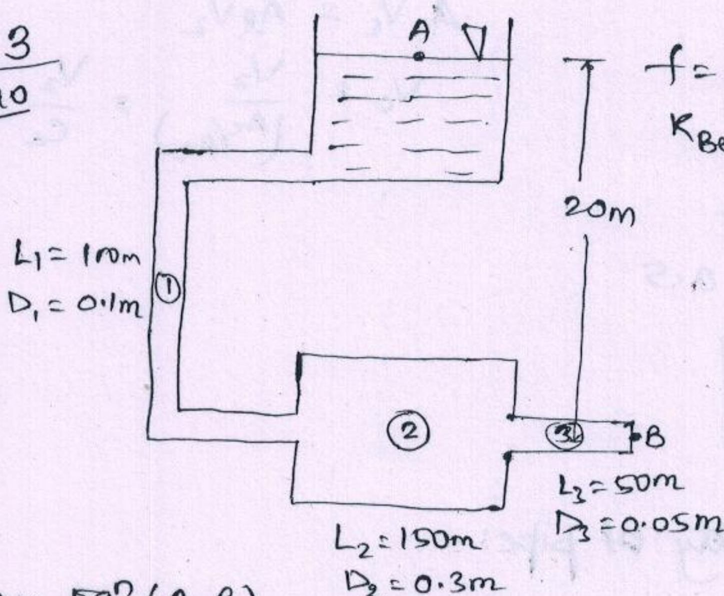
K depends upon

(i) Radius of Pipe if $R_{pipe} \uparrow \Rightarrow K \downarrow$

(ii) Angle of Bend if $\theta \uparrow \Rightarrow K \uparrow$

(iii) Radius of Bend if $R_{Bend} \uparrow \Rightarrow K \downarrow$

Ques 3
Pg 40



$f = 0.008$
$K_{Bend} = 1$

Energy Eqⁿ (A, B).

$$\cancel{\frac{P_{atm}}{\rho g}} + \cancel{\frac{V_A^2}{2g}}^{0} + 20 = \cancel{\frac{P_{atm}}{\rho g}} + \cancel{\frac{V_B^2}{2g}}^{0} + 0 + h_L$$

$$\boxed{h_L = 20}$$

Actual Analysis.

$$h_L = 20 = \frac{(0.008)(100)V_1^2}{2\times9.81\times0.1} + \frac{(0.008)(150)V_2^2}{2\times9.81\times0.3} + \frac{(0.008)(50)V_3^2}{2\times9.81\times0.05}$$

$$+ \frac{0.5V_1^2}{2g} + \frac{V_1^2}{2g} + \frac{V_1^2}{2g} + \frac{(V_1-V_2)^2}{2g} + \frac{(0.5)V_3^2}{2g} + \frac{V_3^2}{2g} \quad \text{—(1)}$$

Continuity Eqⁿ.

$$\frac{\pi}{4}(0.1)^2V_1 = \frac{\pi}{4}(0.3)^2V_2 = \frac{\pi}{4}(0.05)^2V_3.$$

$$4V_1 = 36V_2 = V_3$$

$V_1 = 9V_2$
$V_3 = 36V_2$

$V_2 = 0.172$ m/s

$$Q_{act} = \frac{\pi}{4}(0.3)^2V_2 = 0.0121\ m^3/s$$

Approximate Analysis.

$$h_L = 20 = \frac{(0.008)(100)V_1^2}{2\times9.81\times0.1} + \frac{(0.008)(150)V_2^2}{2\times9.81\times0.3} + \frac{(0.008)(50)V_3^2}{2\times9.81\times0.05}$$

$V_2 = ?$ 0.188 m/s

$$Q_{Appro} = \frac{\pi}{4}(0.3)^2V_2 = 0.0133\ m^3/s$$

$$\%\ Error = \left(\frac{0.0133 - 0.0121}{0.0121}\right)\times100$$

$$= 9.91\ \%$$

Total Energy Line :- (Energy Gradient Line) (TEL/EGL)

It is the line joining the points representing the values of total Head at the various cross section of the pipe in a pipe flow.

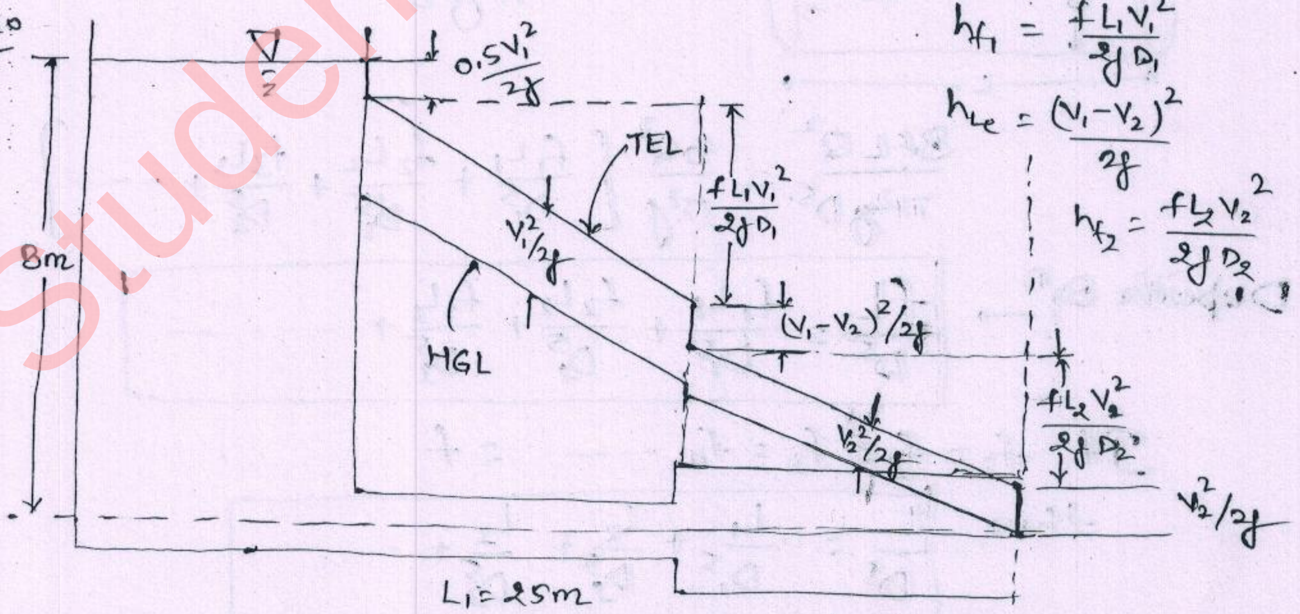
- This line always goes down in the dirn of flow until or unless some energy is supplied externally.

Hydraulic Gradient Line (HGL)

It is the line joining a points representing values of piezometric heads at the various cross section of the pipes in pipe flow.

- This line may go up and down in the dirn of fluid flow
- This line will always be below Total Energy Line (TEL)
- In a Uniform dia pipe this line will always be parallel to total Energy line and the vertical gap b/w these two lines at any section of the pipe represents the value of Kinetic Head.

Ques 4
Pg 40



$h_{L_{entry}} = \frac{0.5V_1^2}{2g}$

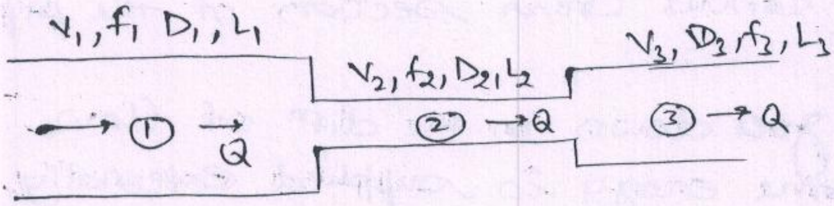
$h_{f_1} = \frac{fL_1V_1^2}{2gD_1}$

$h_{L_e} = \frac{(V_1-V_2)^2}{2g}$

$h_{f_2} = \frac{fL_2V_2^2}{2gD_2}$

$h_{L_{exit}} = \frac{V_2^2}{2g}$

Series Connection of pipes



total Head loss

$$h_f = h_{f_1} + h_{f_2} + h_{f_3} + h_{f_4} \text{ ———}$$

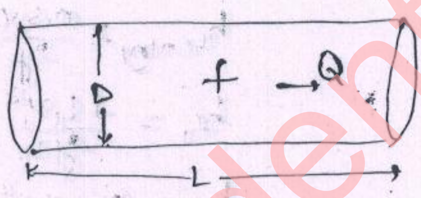
$$h_f = \frac{fLV^2}{2gD} \qquad V = \frac{Q}{\frac{\pi}{4}D^2} = \frac{4Q}{\pi D^2} = \frac{8fLQ^2}{\pi^2 g D^5}$$

$$\text{so } h_f = \frac{8f_1L_1Q^2}{\pi^2 g D_1^5} + \frac{8f_2L_2Q^2}{\pi^2 g D_2^5} + \text{———}$$

$$h_f = \frac{8Q^2}{\pi^2 g}\left[\frac{f_1L_1}{D_1^5} + \frac{f_2L_2}{D_2^5} + \frac{f_3L_3}{D_3^5} + \text{———}\right]$$

Equivalent pipe

that pipe which gives same discharge under same head loss



$$h_f = \frac{8fLQ^2}{\pi^2 g D^5}$$

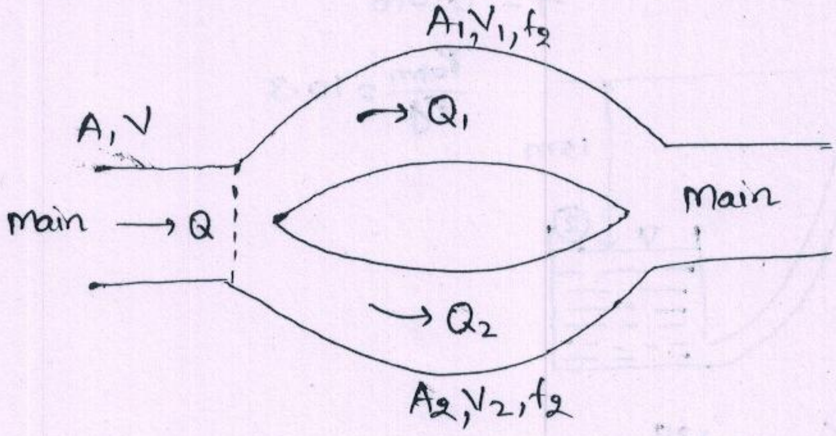
$$\frac{8fLQ^2}{\pi^2 g D^5} = \frac{8Q^2}{\pi^2 g}\left[\frac{f_1L_1}{D_1^5} + \frac{f_2L_2}{D_2^5} + \frac{f_3L_3}{D_3^5} + \text{———}\right]$$

Dupuit's Eq^n $\rightarrow$ $\boxed{\frac{fL}{D^5} = \frac{f_1L_1}{D_1^5} + \frac{f_2L_2}{D_2^5} + \frac{f_3L_3}{D_3^5} + \text{———}}$

If $f_1 = f_2 = f_3 = f_4 \text{ ———} = f$

then $\boxed{\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5} + \text{———}}$

Parallel Connection of pipes:-



$$Q = Q_1 + Q_2$$

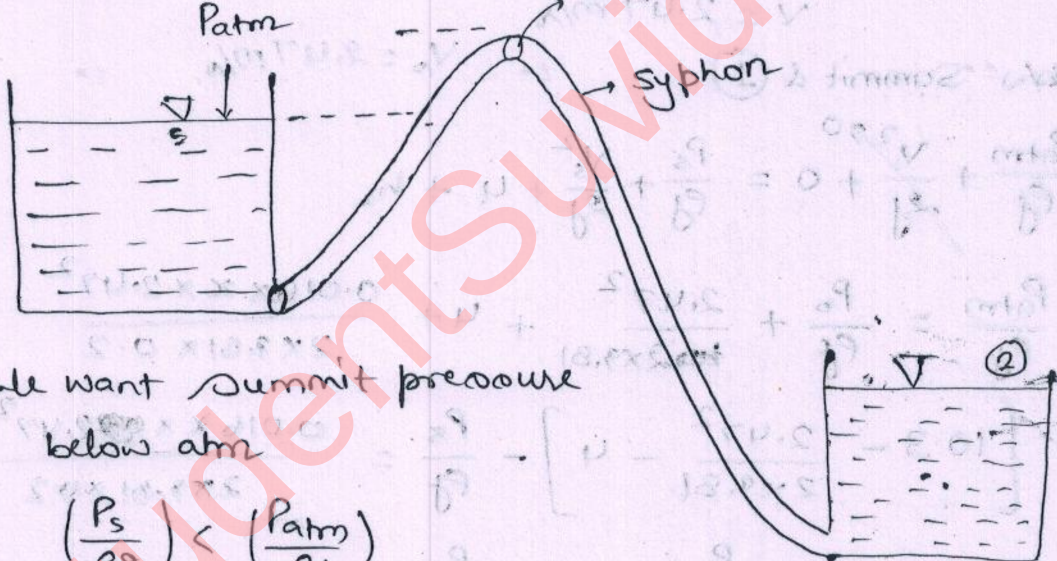
$$\boxed{AV = A_1V_1 + A_2V_2}$$

$$h_{f_1} = h_{f_2}$$

$$\boxed{\frac{f_1 L_1 V_1^2}{2g D_1} = \frac{f_2 L_2 V_2^2}{2g D_2}}$$

Flow through Syphons:-

It is a long bend tube which is basically used to transfer the fluid from one reservoir to the other reservoir.



We want summit pressure below atm

$$\left(\frac{P_s}{\rho g}\right) < \left(\frac{P_{atm}}{\rho g}\right)$$

$$\left(\frac{P_{vapor}}{\rho g}\right)_{water} = (2.7 - 3)\,m \text{ water}$$

$$\left(\frac{P_s}{\rho g}\right)_{Min} = \left(\frac{P_v}{\rho g}\right)$$

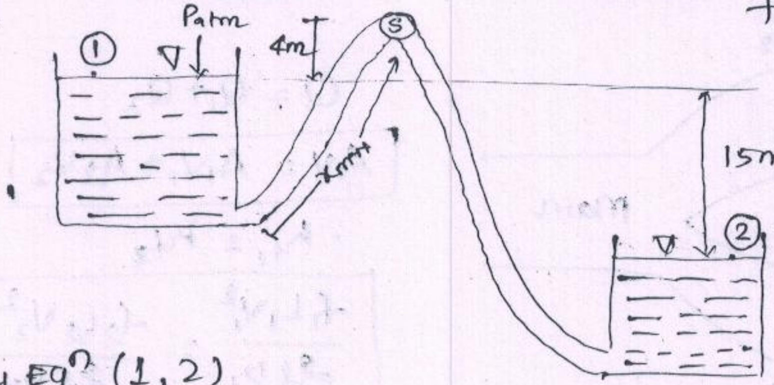
Ques 6
Pg 40

$D = 0.2m$
$L = 600m$

$\left(\frac{P_v}{\rho g}\right)_{water} = 2.8m\,(water)$

$f = 0.016$

$\frac{P_{atm}}{\rho g} = 10.3$



Energy Eq^n (1, 2)

$$\cancel{\frac{P_{atm}}{\rho g}} + \cancel{\frac{V_1^2}{2g}}^{\,0} + 15 = \cancel{\frac{P_{atm}}{\rho g}} + \cancel{\frac{V_2^2}{2g}}^{\,0} + 0 + h_L$$

$$\boxed{h_L = 15\,m}$$

$$15 = \frac{fLV^2}{2gD} = \frac{(0.016) \times 600 \times V^2}{2 \times 9.81 \times 0.2}$$

$$V = 2.47\,m/s$$

$V_s = 2.47\,m/s$

Energy Eq^n b/w Summit & ①

$$\frac{P_{atm}}{\rho g} + \cancel{\frac{V_1^2}{2g}}^{\,0} + 0 = \frac{P_s}{\rho g} + \frac{V_s^2}{2g} + 4 + h_L$$

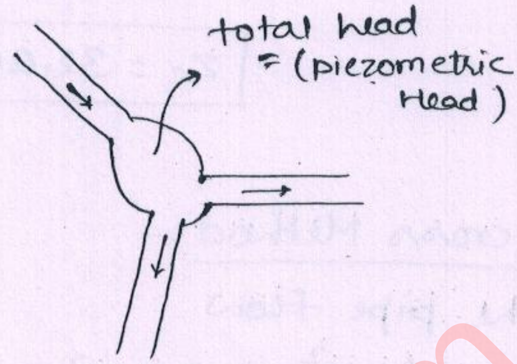
$$\frac{P_{atm}}{\rho g} = \frac{P_s}{\rho g} + \frac{2.47^2}{2 \times 9.81} + 4 + \frac{0.016 \times x \times 2.47^2}{2 \times 9.81 \times 0.2}$$

$$\left[10.3 - \frac{2.47^2}{2 \times 9.81} - 4\right] - \frac{P_s}{\rho g} = \frac{0.016 \times x \times 2.47^2}{2 \times 9.81 \times 0.2}$$

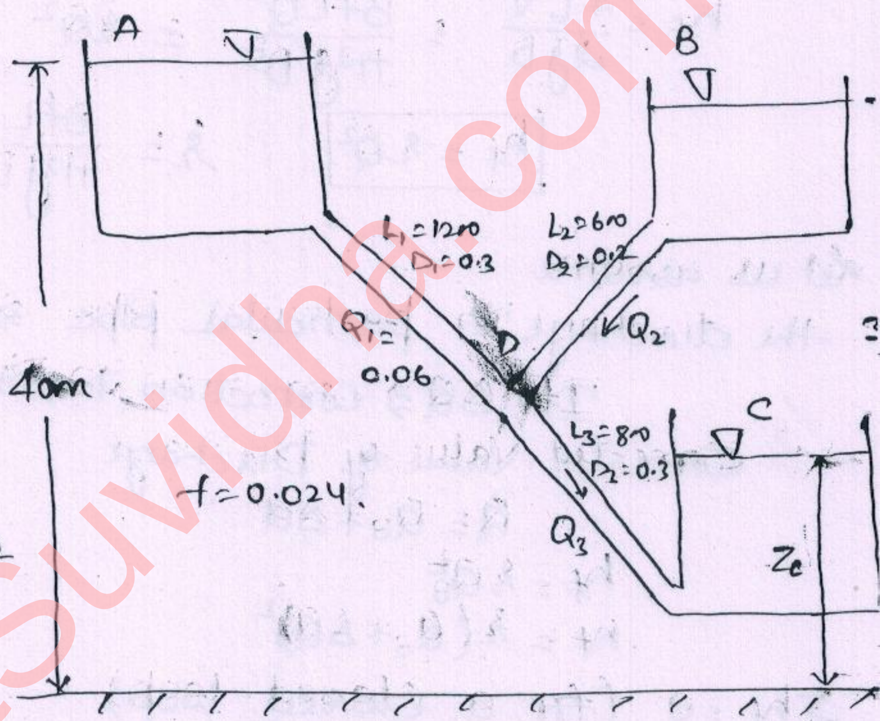
$$\frac{P_s}{\rho g} = 2.8 = \frac{P_v}{\rho g}$$

$$\boxed{x = 128.2\ meter}$$

Flow through pipe junctions:- (Concept of Branched pipe Network)



Ques 7
Pg 40



H = total Head

$$V_1 = \frac{0.06}{\frac{\pi}{4} \times 0.3^2}$$

$$V_1 = 0.849 \, m/s$$

$$H_A = 10.3 + 40 = 50.3 m$$

$$H_A - H_D = h_{f_1}$$

$$= \frac{0.024 \times 1200 \times V_1^2}{2 \times 9.81 \times 0.3}$$

$$H_D = 46.773 m$$

$$H_B = 10.3 + 38 = 48.3 m$$

$$H_B - H_D = h_{f_2} = \frac{0.024 \times 600 \times V_2^2}{2 \times 9.81 \times 0.2}$$

$$V_2 = 0.645$$

$$Q_2 = \frac{\pi}{4}(0.2)^2 \times 0.645 = 0.0203 \, m^3/s$$

$$Q_3 = Q_1 + Q_2 = 0.06 + 0.0203 = 0.0803 \, m^3/s.$$

$$V_3 = \frac{0.0803}{\frac{\pi}{4} \times 0.3^2} = 1.136 \, m/s.$$

$$h_{f_3} = \frac{0.024 \times 800 \times V_3^2}{2 \times 9.81 \times 0.30}$$

$$h_{f_3} = 4.209 \, m$$

$$H_D - H_C = 4.209$$

$$46.773 - (10.3 + Z_c) = 4.209$$

$$\boxed{Z_c = 32.264\, m}$$

Hardy-cross Method :-

In the pipe flow

$$h_f = \frac{fLV^2}{2gD} = \frac{8fLQ^2}{\pi^2 g D^5} = rQ^2$$

$$\boxed{h_f = rQ^2} \qquad r = \frac{8fL}{\pi^2 g D^5} = \text{constant for a pipe}$$

Let us assume

the discharge in particular pipe $\Rightarrow Q_0$ (Assumed)

If $\Delta Q \Rightarrow$ correction in Discharge

Corrected value of Discharge

$$Q = Q_0 + \Delta Q$$

$$h_f = rQ_0^2$$

$$h_f = r(Q_0 + \Delta Q)^2$$

$\Sigma h_f = 0$ (for a closed loop)

$$\Sigma r(Q_0 + \Delta Q)^2 = 0.$$

$$\Sigma(rQ_0^2 + 2rQ_0\Delta Q + r\cancel{\Delta Q^2}) = 0$$

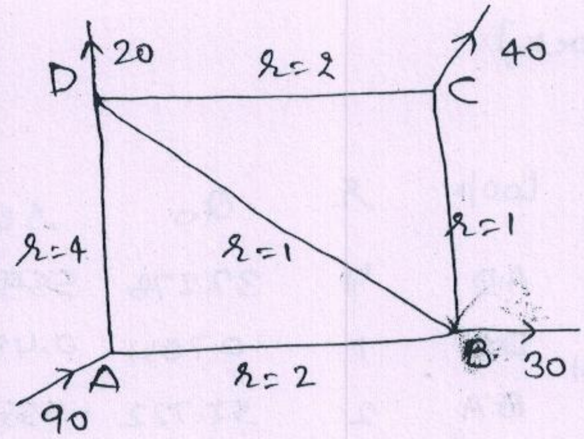
$$\therefore \Sigma rQ_0^2 + \Sigma 2r Q_0 \Delta Q = 0$$

ΔQ is constant for all pipes in a loop

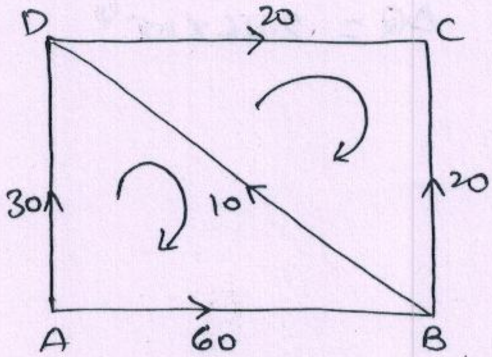
$$\Sigma rQ_0^2 + \Delta Q \Sigma 2rQ_0 = 0$$

$$\boxed{\Delta Q = -\frac{\Sigma rQ_0^2}{\Sigma |2rQ_0|}}$$

Ques



I^{st} Iteration.



loop DCBD

Pipe	r	Q_0	rQ_0^2	$\lvert 2rQ_0 \rvert$
DC	2	20	800	80
CB	1	20	-400	40
BD	1	10	100	20

$\Sigma rQ_0^2 = 500 \quad \Sigma \lvert 2rQ_0 \rvert = 140$

$$\Delta Q = -\frac{500}{140} = -3.571$$
$$= 3.571 (AC)$$

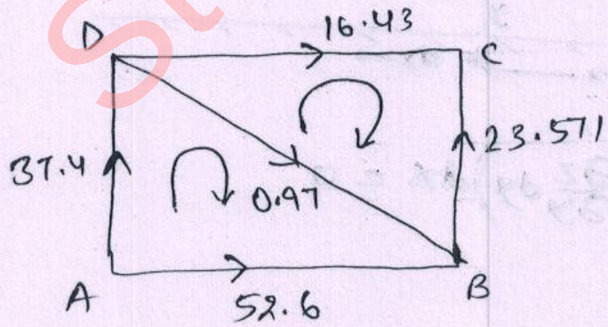
loop ADBA

Pipe	r	Q_0	rQ_0^2	$\lvert 2rQ_0 \rvert$
AD	4	30	3600	240
DB	1	10	-100	20
BA	2	60	-7200	240

$\Sigma rQ_0^2 = -3700, \ \Sigma 2rQ_0 = 500.$

$$\Delta Q = -\frac{\Sigma rQ_0^2}{\Sigma \lvert 2rQ_0 \rvert} = \frac{3700}{500} = 7.4$$ (clock

After Applying Correction.



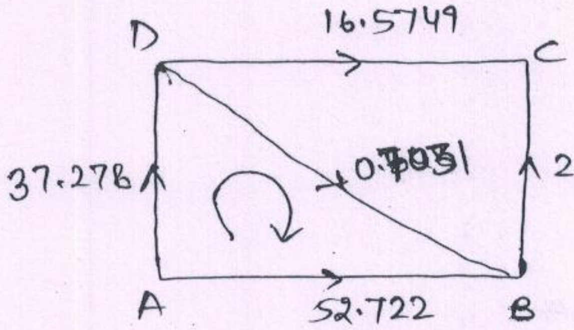
loop ADBA

Pipe	r	Q_0	rQ_0^2	$\lvert 2rQ_0 \rvert$
AD	4	37.4	5595.04	299.2
DB	1	0.97	0.9409	1.94
BA	2	52.6	-5533.52	210.4
			62.4609	511.54

$$\boxed{\Delta Q = -0.122}$$

loop DCBD = 0.1449 (clock)

3rd Iteration



loop	r	Q_0	rQ_0^2	$\lvert 2rQ_0 \rvert$
AD	4	37.278	5558.60	298.22
DB	1	0.7031	0.4943	1.4062
BA	2	52.722	−5559.22	210.88
			−0.1257	510.57

$\Delta Q = 2.46 \times 10^{-4}$

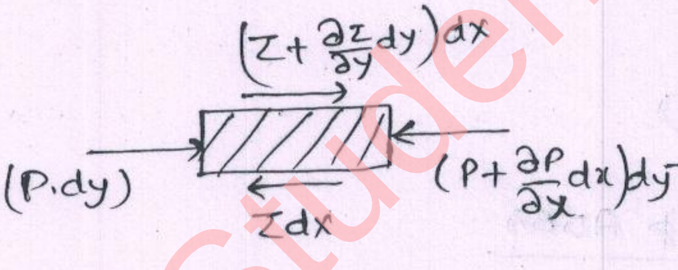
loop DCBD = 0.0015 (AC)

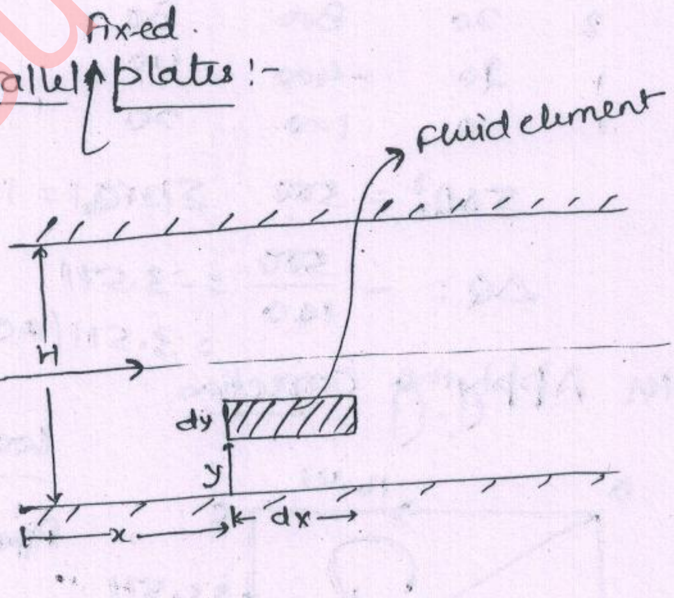
Laminar flow through pipes :-

Incompressible viscous fluid flow

Internal laminar flow

Laminar flow between two parallel plates :-





In the dirn of flow

$\vec{F} = 0$

$$P dy - \left(P + \frac{\partial P}{\partial x} dx\right) dy - \tau dx + \left(\tau + \frac{\partial \tau}{\partial y} dy\right) dx = 0$$

$$\boxed{\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial x}}$$

$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) = \frac{\partial P}{\partial x}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) = \frac{1}{\mu}\left(\frac{\partial P}{\partial x}\right)$$

Integrating $\quad \frac{\partial u}{\partial y} = \frac{y}{\mu}\left(\frac{\partial P}{\partial x}\right) + C_1$

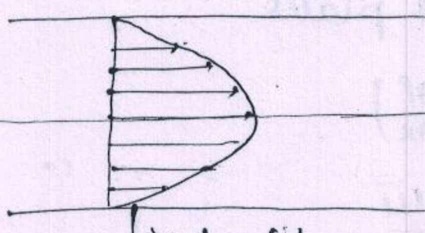
Integrating $\quad u = \frac{y^2}{2\mu}\frac{\partial P}{\partial x} + C_1 y + C_2$

At $y=0 \quad u=0 \qquad C_2 = 0$

and $y=H, u=0 \qquad C_1 = -\frac{H}{2\mu}\left(\frac{\partial P}{\partial x}\right)$

$$u = \frac{y^2}{2\mu}\left(\frac{\partial P}{\partial x}\right) - \frac{yH}{2\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$\boxed{u = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)(Hy - y^2)}$$ Velocity profile (parabolic)



u_{max}

At $y = \frac{H}{2}$, $u = u_{max}$

$$u_{max} = -\frac{1}{2\mu}\left(\frac{\partial P}{\partial x}\right)\left[\frac{H^2}{2} - \frac{H^2}{4}\right]$$

$$\boxed{u_{max} = -\frac{H^2}{8\mu}\frac{\partial P}{\partial x}}$$

Mean Velocity ($\bar{u}$, V)

that Constant velocity at the section which crosses the same mass flow rate as original was crossing is known as Mean Velocity.

$$\dot{m} = \int_0^H \rho u (dy.1) = \rho (H \times 1)\bar{u}$$

$$\bar{u} = \frac{1}{H}\int_0^H u\,dy = \frac{1}{H}\left(-\frac{1}{2\mu}\right)\left(\frac{\partial P}{\partial x}\right)\int_0^H (Hy - y^2)\,dy$$

$$\bar{u} = -\frac{1}{2\mu}\cdot\frac{1}{H}\cdot\frac{\partial P}{\partial x}\left[\frac{H^3}{6}\right] = -\frac{H^2}{12\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$\boxed{\bar{u} = V = -\frac{H^2}{12\mu}\left(\frac{\partial P}{\partial x}\right)} \qquad \frac{\bar{u}}{u_{max}} = \frac{8}{12} = \frac{2}{3}$$

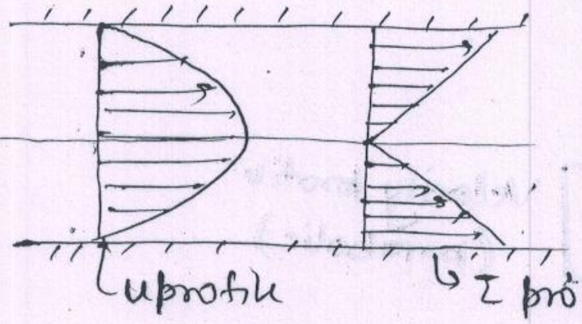
$$\boxed{\bar{u} = \frac{2}{3}u_{max}}$$

Shear stress distribution

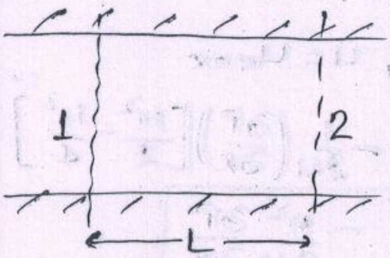
$$\tau = \mu\left(\frac{\partial u}{\partial y}\right)$$

$$\tau = \mu\left(-\frac{1}{2\mu}\right)\left(\frac{\partial p}{\partial x}\right)[H-2y]$$

$$\boxed{\tau = -\frac{1}{2}\left(\frac{\partial p}{\partial x}\right)(H-2y)}$$ linear profile



Head loss in the flow b/w parallel plates



$$\bar{u} = -\frac{H^2}{12\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$-\frac{\partial p}{\partial x} = \frac{12\mu\bar{u}}{H^2}$$

$$-\int_1^2 \partial p = \frac{12\mu\bar{u}}{H^2}\int_0^L dx$$

$$P_1 - P_2 = \frac{12\mu\bar{u}L}{H^2}$$

in Head form $\frac{P_1-P_2}{\rho g} = \frac{12\mu\bar{u}L}{\rho g H^2}$

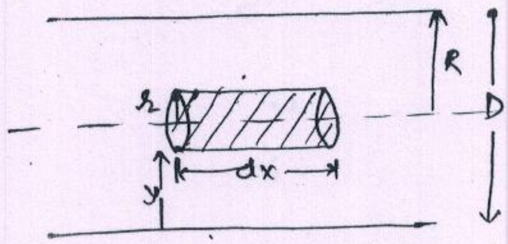
loss of total head $\longrightarrow$ $\boxed{h_{f_L} = \frac{12\mu\bar{u}L}{\rho g H^2}}$

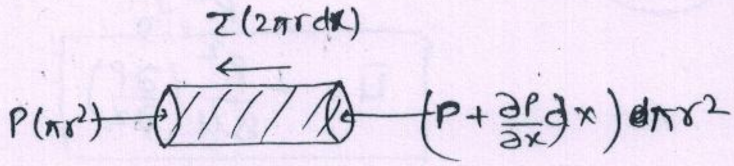
Laminar flow b/w two parallel fixed plate

1 $u = -\frac{1}{2\mu}\frac{\partial p}{\partial x}[Hy - y^2]$

2 $\bar{u} = V = \frac{2}{3}u_{max}$

Laminar flow through pipes:-





In the dirn of flow $\vec{F} = 0$

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x} dx\right)\pi r^2 - \tau(2\pi r\, dx) = 0$$

$$\boxed{\tau = -\frac{r}{2}\left(\frac{\partial P}{\partial x}\right)}$$ Linear profile

$$\mu \frac{\partial u}{\partial y} = -\frac{r}{2}\left(\frac{\partial P}{\partial x}\right)$$

$y + r = R$
$dy + dr = 0$
$dy = -dr$

$$\mu \frac{\partial u}{-\partial r} = -\frac{r}{2}\left(\frac{\partial P}{\partial x}\right)$$

$$\frac{\partial u}{\partial r} = \frac{r}{2\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$u = \frac{r^2}{4\mu}\left(\frac{\partial P}{\partial x}\right) + c$$

at $r = R$, $u = 0$

$$c = -\frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)$$

$$u = \frac{r^2}{4\mu}\frac{\partial P}{\partial x} - \frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)$$

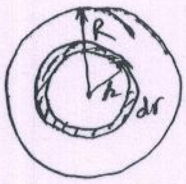
$$\boxed{u = -\frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)(R^2 - r^2)}$$ Parabolic Distribution

u_{max}: At $r = 0$ $u = u_{max}$

$$\boxed{u_{max} = -\frac{R^2}{4\mu}\left(\frac{\partial P}{\partial x}\right)}$$

Mean Velocity $(\bar{u}, \bar{v})$

$$\dot{m} = \int_0^R \rho u (2\pi r dr) = \rho(\pi R^2)\bar{u}$$

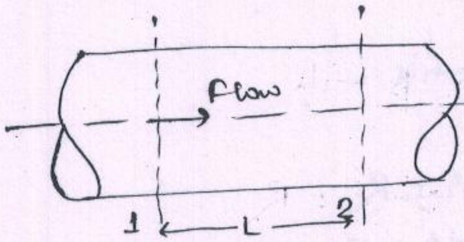


$$\bar{u} = \frac{2}{R^2}\int_0^R u r dr = \frac{2}{R^2}\left(-\frac{1}{4\mu}\right)\left(\frac{\partial p}{\partial x}\right)\int_0^R (R^2 r - r^3) dr$$

$$\boxed{\bar{u} = -\frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)}$$

$$\frac{\bar{u}}{u_{max}} = \frac{1}{2} \Rightarrow \boxed{\bar{u} = \frac{1}{2} \times u_{max}}$$

Head loss in pipe flow



$$\bar{u} = -\frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right) = -\frac{D^2}{32\mu}\left(\frac{\partial p}{\partial x}\right)$$

$$-\int_1^2 \partial p = \frac{32\mu\bar{u}}{D^2}\int_0^L dx$$

$$(P_1 - P_2) = \frac{32\mu\bar{u}L}{D^2}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{32\mu\bar{u}L}{\rho g D^2}$$

$Re_D < 2000$ $\boxed{h_f = \frac{32\mu\bar{u}L}{\rho g D^2}}$ loss of total head → Hagen-poiseuille.

Only valid in laminar flow through pipes

Friction factor in laminar flow through pipes

$$h_f = \frac{32\mu\bar{u}L}{\rho g D^2} = \frac{fLV^2}{2gD} \quad (\because \bar{u} = V)$$

$$f = \frac{64\mu}{\rho V D} = \frac{64}{\left(\frac{\rho V D}{\mu}\right)} = \frac{64}{Re_D}$$

$$\boxed{f = \frac{64}{Re_D}}$$

aminar flow through pipes

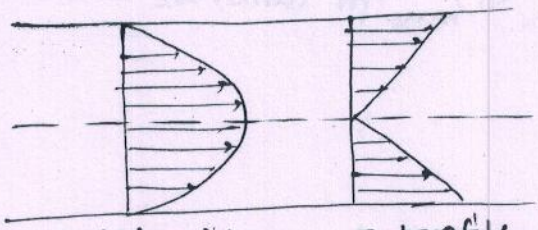
) $u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2)$ 2.) $\bar{u} = \frac{1}{2}u_{max}$ 3.) $h_f = \frac{fLV^2}{2gD} = \frac{32\mu\bar{u}L}{\rho g D^2}$

Concept of Shear Velocity (V^*) in pipes:-

In laminar flow

shear stress distribution

$$\tau = -\frac{r}{2}\left(\frac{\partial P}{\partial x}\right)$$



surface shear stress

at $y=0 \Rightarrow r = R$

$\tau = \tau_0$

$$\tau_0 = -\frac{R}{2}\left(\frac{\partial P}{\partial x}\right)$$

$$\boxed{\tau_0 = -\frac{D}{4}\left(\frac{\partial P}{\partial x}\right)}$$

$$-\int_1^2 \partial P = \int_0^L \frac{4\tau_0}{D}\,dx \Rightarrow P_1 - P_2 = \frac{4\tau_0 L}{D}$$

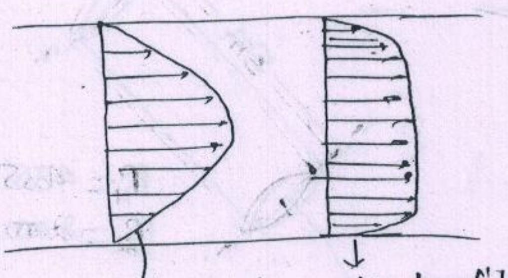
$$\boxed{\frac{P_1 - P_2}{\rho g} = h_f = \frac{4\tau_0 L}{\rho g D}}$$

$$h_f = \frac{fLV^2}{2gD} = \frac{4\tau_0 L}{\rho g D}$$

$$\frac{\tau_0}{\rho} = \frac{f}{8}V^2$$

V^* is same in laminar & turbulent

$$\boxed{\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}} \times V^*}$$



Ques Show that the Max^m shear stress on the pipe wall for laminar flow through pipe of Dia D with given fluid properties is $\frac{16000\mu^2}{\rho D^2}$

Ans Shear stress on pipe wall τ_0

$$h_f = \frac{4\tau_0 L}{\rho g D} = \frac{fLV^2}{2gD}$$

$$\tau_0 = \frac{\rho f V^2}{8} = \frac{64}{8\,Re_D}\rho V^2$$

$$Re_D = \frac{\rho V D}{\mu} \Rightarrow V = \frac{Re_D\,\mu}{\rho D}$$

$$\tau_0 = \frac{8 Re_D \mu^2}{\rho D^2}$$

$$\tau_{0max} = \frac{8 (Re_D)_{max} \mu^2}{\rho D^2} \quad (Re_D)_{max} \text{ for laminar} = 2000$$

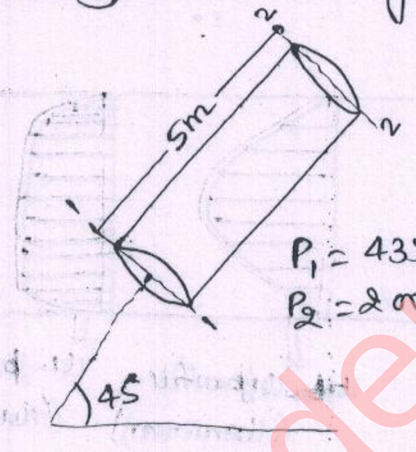
$$\boxed{\tau_{0max} = \frac{16000 \mu^2}{\rho D^2}}$$

Ques. An upward flow of oil [(800 kg/m³), (0.8 Pas)] takes plates in laminar condition in an inclined pipe of 10 cm dia as shown in fig. the pressure at section 1 and section 2 are 435 and 200 kPa respectively. Find

(i) Discharge through the pipe.

(ii) If the flow dirn are reversed what should be the pressure required at the section 2 for the same discharge while by maintaining the pressure of section 1 as it is.

Ans



$P_1 = 435$ kPa

$P_2 = 200$ kPa

$V_1 = V_2$

$D = 0.1$ m

(i) Flow is $1 \rightarrow 2$.

$$h_f = H_1 - H_2.$$

$$= \left(\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g}\right) - \left(\frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}\right)$$

$$= \frac{(435 - 200)\times 10^3}{800 \times 9.81} - (z_2 - z_1)$$

$$\frac{32 \mu \bar{u} L}{800 \times 9.81 \times (0.1)^2} = \left(\frac{235 \times 10^3}{800 \times 9.81} - 5 \sin 45^\circ\right) = 26.4084$$

$$\bar{u} = 16.19 \text{ m/s}$$

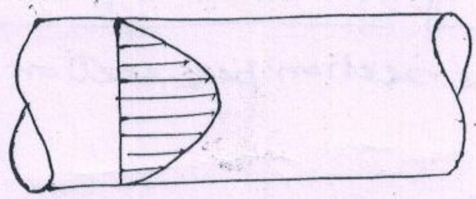
$$Q = \frac{\pi}{4}(0.1)^2 \times 16.19 = 0.127 \text{ m}^3/\text{s}$$

(ii) Flow is 2 – 1

$$\left(\frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}\right) - \left(\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g}\right) = 26.4084$$

$$\frac{P_2 - P_1}{\rho g} + 5 \sin 45^\circ = 26.4084 \Rightarrow \boxed{P_2 = 614.5063 \text{ kPa}}$$

Laminar flow through pipes



$$u = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)(R^2 - r^2)$$

$$\bar{u} = -\frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right).$$

Momentum Correction factor (β)

$$\beta = \frac{\text{Actual Momentum Crossing through the section per sec}}{\text{Momentum crossing through section per sec based on Mean Velocity.}}$$

$$\beta = \frac{\int_0^R \rho u (2\pi r) dr\, u}{\dot{m}\,\bar{u}} = \frac{2\pi\rho \int_0^R u^2 r dr}{\rho(\pi R^2)\bar{u}\cdot\bar{u}}$$

$$\beta = \frac{2}{R^2} \times \frac{1}{\bar{u}^2}\int_0^R u^2 r dr$$

$$\beta = \frac{\frac{2}{R^2} \times \left(-\frac{1}{4\mu}\right)^2 \left(\frac{\partial p}{\partial x}\right)^2 \int_0^R (R^2 - r^2) r dr}{\left(-\frac{R^2}{8\mu}\right)^2 \left(\frac{\partial p}{\partial x}\right)^2}$$

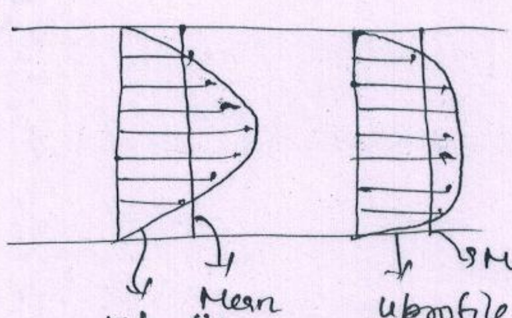
$$\beta = \frac{8}{R^6}\int_0^R (R^4 r + r^5 - 2R^2 r^3) dr$$

$$\beta = \frac{8}{R^6}\left[\frac{R^6}{2} + \frac{R^6}{6} - \frac{R^6}{2}\right] = \frac{8}{6}$$

$$\boxed{\beta = 1.33}$$

$\beta_{\text{laminar flow (pipes)}} = 1.33$

$\beta_{\text{turbulent flow (pipes)}} = 1.2$



Kinetic Energy Correction factor (α)

$$\alpha = \frac{\text{Actual Kinetic Energy crossing through the section per sec}}{\text{Kinetic Energy crossing through the section per section based on mean velocity}}$$

$$\alpha = \frac{\frac{1}{2}\int_0^R \rho u(2\pi r dr) u^2}{\frac{1}{2}(\dot{m}\bar{u}^2)} = \frac{2\pi\rho\int_0^R u^3 r dr}{\rho(\pi R^2)\bar{u}\,\bar{u}^2}$$

$$\alpha = \frac{\frac{2}{R^2}\int_0^R u^3 r dr}{\bar{u}^3} = \frac{\frac{2}{R^3}\left(-\frac{1}{4\mu}\right)^3\left(\frac{\partial p}{\partial x}\right)^3\int_0^R (R^2-r^2)^3 r dr}{\left(-\frac{R^2}{8\mu}\right)^3\left(\frac{\partial p}{\partial x}\right)^3}$$

$$\alpha = \frac{16}{R^8}\int_0^R (R^2-r^2)^3 r dr$$

$$\alpha_{laminar} = 2$$

$$\alpha_{turbulent} = 4/3 = 1.33$$

β → laminar = 1.33, → turbulen = 1.2

α → laminar = 2, → turbulent = 1.33

Energy Eq^n $\boxed{p + \frac{1}{2}\rho V^2 + \rho g Z = \text{Const}}$

V = Avg.

DIMENSIONAL ANALYSIS.

AIM:-

1) Development of functional Relationships:-

Rayleigh's Method:-

Ques the time period of simple pendulum depends upon the length of the simple pendulum and accelaration due to gravity find the expression for the time period of simple pendulum.

Ans.

$$T \propto L^a g^b$$

$$[T] = C[L]^a[LT^{-2}]^b$$

$$[T] = C[L^{a+b}\,T^{-2b}]$$

$$a+b=0 \qquad -2b=1$$

$$b=-\tfrac{1}{2} \text{ and } a=\tfrac{1}{2}$$

$$T = C L^{1/2} g^{-1/2}$$

$$T = C\sqrt{\frac{L}{g}}$$

By experiments $C = 2\pi$

$$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$$

Ques $\underbrace{F_D}_{\text{Dependent}}$ $\quad \underbrace{D, V, \rho\, \mu}_{\text{Independent Variable}}$

$$F_D \propto D^a V^b \rho^c \mu^d$$

$$[MLT^{-2}] = C[L]^a[LT^{-1}]^b[ML^{-3}]^c[M^1L^{-1}T^{-1}]^d$$

$$[MLT^{-2}] = C[L^{a+b-3c-d}\,M^{c+d}\,T^{-b-d}]$$

$$c+d=1$$

$$a+b-3c-d=1$$

$$-b-d=-2$$

$$b=2-d$$

$$c=1-d$$

$$a+2-d-3+3d-d=1$$

$$a=2-d$$